YOUNG DUAL
ARITHMETICIAN

OLIVER BYRNE.

THE

YOUNG DUAL ARITHMETICIAN.

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THE

Young Dual Arithmetician;

OR,

DUAL ARITHMETIC.

A NEW ART, DESIGNED FOR ELEMENTARY INSTRUCTION AND THE USE OF SCHOOLS.

TO WHICH ARE ADDED,

TABLES OF ASCENDING AND DESCENDING DUAL LOGARITHMS, DUAL NUMBERS, AND CORRESPONDING NATURAL NUMBERS.

Second Edition,

REVISED AND AMENDED.

BY

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INVENTOR OF THE ART AND SCIENCE OF DUAL ARITHMETIC; AND THE CALCULUS OF FORM, A NEW MATHEMATICAL SCIENCE.

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1871.

PREFACE.

THOSE who examine Dual Arithmetic in all its bearings will find that a branch of greater importance has not been contributed to mathematical science. Important as this branch of mathematics is, it might be long neglected for the want of a treatise of a nature sufficiently elementary, which requirement this work is designed to supply. Although this school-book is suited for young students of different capacities, yet it contains many concise rules and contracted processes not to be found in my larger works on the Art and Science of Dual Arithmetic. However, I have maintained throughout the present work a method of teaching that approaches nearest the method of investigation which I have pursued in my other works on the same subject, because such procedure possesses many advantages in extending elementary instruction. In teaching the elements of a GREAT ART like Dual Arithmetic, entirely new, many examples are necessary, yet a healthy impression should be left on the mind of a scholar, and the spirit of inquiry strengthened and not disgusted by monotonous repetitions. The following operative numbers, namely,

1·
1·1
1·2·1
1·3·3·1·
1·4·6·4·1·
1·5·10·10·5·1·

are of great use in Dual Arithmetic; and although these

numbers are readily formed (see pp. 21, 35) and easily remembered, yet I make special reference to them here to impress their importance on the mind of the student.

It may be further necessary to inform the reader that I discovered the Art and Science of Dual Arithmetic, upon which I have written elementary works, entitled, "The Young Dual Arithmetician;" "Dual Arithmetic, a New Art;" "Dual Arithmetic, a New Art, Part the Second." This work is on the descending branch of the art, and treats of the Science of Dual Arithmetic. Part the Third is in the press. Besides, I have published works entitled, "Limited Tables of Dual Logarithms, Angular Magnitudes, and Trigonometrical Lines;" and "General Method of Solving Equations of all Degrees." I have applied Dual Arithmetic to many important inquiries in my "Essential Elements of Practical Mechanics," and in my Dictionary of Civil, Mechanical, Military, and Naval Engineering, which work is called "Spons' Dictionary," after the Publishers, the Messrs. Spon.

OLIVER BYRNE.

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The Young Dual Arithmetician.

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DECIMAL ARITHMETIC

AND

OTHER PRELIMINARY OPERATIONS.

PREVIOUS to commencing the study of Dual Arithmetic, it is necessary to have a clear and philosophical view of Decimal Arithmetic.

OF DECIMAL FRACTIONS.

1. A fraction expresses part of any whole thing by two numbers, one placed above a line and the other below it; the number below the line is called the denominator, and shows how many parts the quantity is to be divided into; the number above the line is called the numerator, and specifies the number of such parts to be taken:—for example, let us take the following fractions,

$$\frac{1}{2};\ \frac{3}{4};\ \frac{2}{3},\ \text{and}\ \frac{4}{7};\ \begin{matrix}\text{numerator,}\\ \text{denominator;}\end{matrix}$$

which are called one-half, three-fourths; two-thirds, and four-sevenths respectively, that is, if a quantity be divided into two equal parts and one of them taken, it is expressed by $\frac{1}{2}$, or one-half; if a quantity be divided into seven equal parts, and four of them taken, it is expressed by $\frac{4}{7}$, and is called four-sevenths, and so of others.

2. When the numerator and denominator are equal, as $\frac{7}{7}$, the fraction is equal one, and when the numerator is greater than the denominator, the fraction is called improper, and is greater than one; $\frac{9}{7}$, which is equal $\frac{7}{7}$ and $\frac{2}{7}$ together, or equal $1\frac{2}{7}$.

3. If the numerator and denominator of any fraction be multipied or divided by any number, it neither increases nor diminishes the value of the fraction; thus,

$$\frac{1}{2}=\frac{2}{4}=\frac{4}{8}=\frac{40}{80}=\frac{400}{800}=\&c.,$$

$$\text{and } \frac{40}{80}=\frac{20}{40}=\frac{2}{4}\quad\frac{1}{2};$$

for it is *evident that* if a quantity be divided into 800 equal

parts, and 400 of them taken, one-half the quantity is taken; the same reasoning holds good with other fractions.

$$\frac{3}{4}=\frac{30}{40}=\frac{300}{400}=\frac{3000}{4000}=\&c.,$$

$$\frac{4}{7}=\frac{40}{70}=\frac{400}{700}=\frac{4000}{7000}=\&c.;$$

so that it appears a fractional part of anything may be expressed in an infinite number of ways.

4. From the above it appears that one fraction may be reduced to another equal to it, and having, either the numerator or denominator, what number we please; for example, let us reduce $\frac{2}{3}$ to a fraction equal to it whose denominator shall be 1000.

$$\frac{2}{3}=\frac{2000}{3000} \text{ and } \frac{2000}{3000}=\frac{666\frac{2}{3}}{1000} \quad \therefore \frac{2}{3}=\frac{666\frac{2}{3}}{1000}$$

5. The reduction of vulgar fractions to decimals, is nothing more than the reduction of one fraction to another whose denominator will be 10, 100, 1000, 10000, &c.; then cancelling the denominator, and placing a full point to the left of the numerator, which must consist of as many places of figures as there are ciphers cancelled in the denominator; any deficiency of figures in the numerator must be made up with ciphers to the left.

Examples in Reduction.

Ex. 1. Reduce $\frac{3}{4}$ to a decimal.

$$\frac{3}{4}=\frac{300}{400} \text{ and } \frac{300}{400}=\frac{75}{100} \quad \therefore \frac{3}{4}=\frac{75}{100} \text{ or } \cdot 75 \text{ of a decimal.}$$

Contracted.

$$4)\underline{30000}$$

$\cdot 75$ *Answer.*

Ex. 2. Reduce 15*s.* 7*d.* to the decimal of a pound sterling.

15*s.* 7*d.* = 187 pence.

£1. = 240

6. Then the vulgar fraction is expressed by $\frac{187}{240}$, the decimal is $\cdot 7791667$ to seven places of figures; this is far enough to continue decimals for most practical purposes, and when the last figure is 5, 6, 7, 8, or 9, the figure *preceding may* be counted one more.

WORK.

```
24|0)187    (77916666, &c.
     168
     ---
      190
      168
      ---
       220
       216
       ---
         40
         24
         --
         160
         144
         ---
          16, &c.
```

IMPORTANT RULE.

☞ When the last figure and all that follow are rejected, and when the last figure is 5, 6, 7, 8, or 9, the figure preceding may be counted one more; ∴ the decimal of $\frac{187}{240}$ to five places is ·77917.

Ex. 3. Reduce $\frac{2}{63}$ to a decimal fraction.

```
63)200000000(·03174603
    189
    ---
     110
      63
     ---
      470
      441
      ---
       290
       252
       ---
        380
        378
        ---
          200
          189
          ---
           11
```

7. In this division there are eight ciphers used, then there must be eight places of figures in the quotient, and therefore one cipher is to be placed to the left of the figures obtained, and then the full point; or, which is the same, for every cipher added to the numerator there should be a cipher or some figure placed in the quotient, and before the figure put in the quotient for the first cipher added, the full point.

Examples for Practice.

4. Reduce 2 feet $7\frac{1}{2}$ inches to the decimal of a yard.
Ans. ·875.

5. What decimal is equal to $\frac{5}{61}$?
Ans. ·081967 and 13 over.

6. Reduce $17\frac{3}{7}$ to a mixed decimal.
Ans. 17·4285714, &c.

To Find the Value of any Decimal.

Rule.—Multiply the given decimal by any denomination less than it, pointing off as many decimals in the product as are in the given decimal; then the figure or figures to the left of the separating point will be the number of that denomination contained in the decimal. Where the decimal has to be reduced to two or more denominations, the process may be repeated, as in the following

Examples.

Ex. 1. Required the value of ·1111111 of a hogshead containing 63 gallons.

$$\begin{array}{r} \cdot 1111111 \\ 63 \\ \hline 3333333 \\ 6666666 \\ \hline 6 \cdot 9999993 \end{array}$$

8. When ·999.... follows the decimal point, the whole number may be increased by one; therefore the answer is 7 gallons.

Ex. 2. What is the value of ·87625 of a pound sterling?

$$\left.\begin{array}{r} \cdot 87625 \\ 20 \\ \hline 17 \cdot 52500 \\ 12 \\ \hline 6 \cdot 30000 \\ 4 \\ \hline 1 \cdot 20000 \end{array}\right\}$$

∴ the value of £·87625 is equal £0. 17*s.* $6\frac{1}{4}$*d.* and $\frac{2}{10}$ of a farthing over.

Ex. 3. Find the value of ·087648 of a yard.

$$\left.\begin{array}{ll} & \cdot 087648 \\ & \underline{\qquad\quad 3} \\ \text{feet} & = 0\cdot 262944 \\ & \underline{\qquad\quad 12} \\ \text{inches} & = 3\cdot 155328 \\ & \underline{\qquad\quad 12} \\ \text{parts} & = 1\cdot 863936 \end{array}\right\}$$

·087648 of a yard = 3 inches 2 parts nearly.

Examples for Practice.

Ex. 4. Required the value of ·074325 of a pound sterling.
Ans. £0. 1*s.* 5¾*d.* 352.

Ex. 5. Reduce ·8383838 of a foot to its equivalent value of inches and parts. *Ans.* 10 in. 0·7272 parts.

Ex. 6. What is the value of ·2648125 of a mile?
Ans. 2 fur. 4 pol. 4 yds. 0 ft. 2 in. 6·24 pts.

Most civilized nations employ the Metric system, of which we shall speak presently.

Addition of Mixed Numbers and Decimal Fractions.

Rule.—Keep the decimal points under each other and proceed as in common addition, observing to point off as many decimals in the sum as there are in the term which contains the greatest number of decimals;

or,

As placing ciphers after decimals does not increase or diminish their value (4·), affix ciphers until each of the given terms consist of the same number of decimals, then proceed as in common addition, pointing the sum as before.

Examples.

Ex. 1. Required the sum of 3·46; 21·347; 1·3; and 24·0049.

	or thus,
3·46	3·4600
21·347	21·3470
1·3	1·3000
24·0049	24·0049
50·1119	50·1119

9. The reason of this rule is obvious, for

$$\begin{aligned} 3·46 &= 3\tfrac{46}{100} = 3\tfrac{4600}{10000} \\ 21·347 &= 21\tfrac{347}{1000} = 21\tfrac{3470}{10000} \\ 1·3 &= 1\tfrac{3}{10} = 1\tfrac{3000}{10000} \\ 24·0049 &= 24\tfrac{49}{10000} = 24\tfrac{49}{10000} \end{aligned}$$

$$\text{Sum} = 49\tfrac{11119}{10000} =$$

$50\tfrac{1119}{10000} = 50·1119$, the same as above.

Examples for Practice.

Ex. 2. What is the sum of 0·075; 11·0712; 171·; 47·1433 and 145·03215. *Ans.* 374·32165.

Ex. 3. Reduce the following fractions to decimals, and find their sum, namely; $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{10}$, $\frac{2}{11}$, $\frac{8}{25}$ and $\frac{7}{12}$. *Ans.* 1·7684848.

Subtraction of Decimals.

Rule.—Place the numbers under each other as in addition, affixing ciphers, if necessary, to make the decimal places equal; then proceed as in common subtraction, pointing off the decimals in the remainder as in addition.

Examples.

What is the difference between 36·47 and 19·3764; 38·4 and 3·84; and 341·365815 and 276·82.

	Ex. 1.	*Ex.* 2.	*Ex.* 3.
From	36·4700	38·40	341·365815
Take	19·3764	3·84	276·820000
Remainder	17·0936	34·56	64·545815

Ex. 4. Required the difference between 8·75 + ·825 and 5·681 + 2·17. *Ans.* 1·724.

Multiplication of Decimals.

Rule.—Multiply the factors as in common multiplication; then point off as many decimals in the product (from the right) as there are in both factors.

☞ 10. When it happens that the figures in the product are less than the decimals in both factors, ciphers must be prefixed to the left of the product to make up the deficiency. ☜

EXAMPLES.

Ex. 1.

Multiply 14·36 by 16·451

```
       16·451
       ------
       1436
      7180
     5744
    8616
   1436
   ---------
Prod.=236·23636
```

Ex. 2.

Multiply ·0473 by ·0847

```
       ·0847
       -----
        3311
       1892
      3784
  ------------
  ·00400631=Prod.
```

11. The above rule for pointing the decimals in the product may be proved as follows:—

$$14·36 = 14\tfrac{36}{100} = \tfrac{1436}{100}$$
$$\text{and } 16·451 = 16\tfrac{451}{1000} = \tfrac{16451}{1000}$$
$$\therefore\ 14·36 \times 16·451 = \tfrac{1436}{100} \times \tfrac{16451}{1000} = \tfrac{23623636}{100000} =$$
$$236\tfrac{23636}{100000} = 236·23636 \text{ as before.}$$

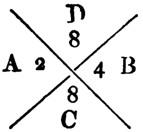
12. As it only occupies a few seconds to prove any multiplication, it is advisable that all should be proved of which any doubt may be entertained.

The following explains the method of proof.

Ex. 3. Let it be required to multiply 23.33 by 16·321 and prove the work to be right or wrong.

```
Multiply 23·33 —(A)
by       16·321—(B)
         ------
         2333
        4666
       6999
     13998
     2333
     ----------
Prod.=380·76893—(C)
```

PROOF.



Cast the nines out of the sum of the digits in (A), and what is over place on the cross to the left, that is, 2+3+3+3=11=one nine and 2 over, which set down; again, cast the nines out of (B), setting down on the cross to the right what is over, that is, 1+6+3=10=one nine and 1 over, then 1+2+1=4, which does not make up nine, then 4 is over, which set down on the cross at (B); then multiply the numbers on the cross at (A) and (B) together, and cast the nines out of the product, and place what is over on the cross at (D).

Lastly cast the nines out of the product (C); thus, 3+8=11=9 and 2 over; 2+7=9 and 0 over; 6+8=14=9 and 5 over; 5+3=8, which place on the cross at (C); then,

if the numbers on the cross at (C) and (D) are equal, the work is right, and if not, wrong. In casting out the nines in any number, the ciphers or nines need not be taken into account.

It is natural to associate the idea of labour with long detail of execution, but such an idea may be abandoned here, as at most it will not take the operator more than half a minute to prove any multiplication by the directions just given.

Examples for Practice.

Ex. 4. Multiply 3·1416 by 10·24. *Ans.* 32·169984.

Ex. 5. Multiply ·00376 by 278. *Ans.* 1·04528.

Ex. 6. Multiply 3·1416+·7854 by 2218·192—277·274.

Ans. 7621·984986 true to six places of decimals.

Division of Decimals.

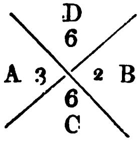
Rule.—Division of decimals is the same as division of whole numbers, only it is to be strictly observed that the number of decimals and ciphers annexed to the dividend must be always equal to the number of decimals both in the divisor and quotient; now, as the number of decimals in the divisor, and also the number of decimals and ciphers affixed to the dividend, are known, the number of decimals in the quotient is determined by making their difference. But as the rule for division of decimals is best drawn from examples, the following are most of the varieties that can occur.

Examples.

Ex. 1. Divide 3647· by 47.

```
                 Dividend.
Divisor  47)3647(77·595744 Quotient.
            329
            ---
             357
             329
             ---
              280              210
              235              188
              ---              ---
               450              220
               423              188
               ---              ---
                270              32 Remainder.
                235
                ---
                 350
                 329
                 ---
                  210
```

13. When the divisor, the dividend made less by the remainder, and the quotient are arranged as below, the proof is the same as in multiplication.



Divisor.	Divid.	Ciphers annexed.	Quotient.
	3647	000000	
		32 Remainder.	
47—(B)	3646·999968—(C)		(77·595744—(A)

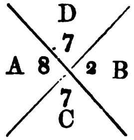
14. When the divisor is a whole number and less than the dividend, another whole number, the quotient is composed of whole numbers and decimals; the number of decimals in the quotient is equal the number of ciphers called down.

Ex. 2. Divide 47· by 3647.

Divisor. Divid. Quotient.
3647·) 47·00 (·012887
3647
10530
7294
32360
29176
31840
29176
26640
25529
1111 Remainder.

PROOF:

Quotient = ·012887—(A)
Divisor = 3647·—(B)



Dividend and ciphers affixed.

From 47·000000
Take 1111 Remainder
46·998889—(C)

15. We obtain five figures in the quotient, but there ought to be six, because there are six ciphers appended to the dividend, and no decimal in the divisor, therefore a cipher must be placed to the left of the figures in the quotient, and then the decimal point.

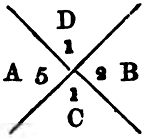
Ex. 3. Divide 347·23 by 5·19878.

```
Divisor.    Dividend.    Quotient.
5·19878 ) 347·2300 ( 66·79067
          3119268
          -------
           3530320
           3119268
           -------
            4110520
            3639146
            -------
             4713740
             4678902
             -------
               3483800
               3119268
               -------
                3645320
                3639146
                -------
                   6174 Remainder.
```

From	The number of decimals and ciphers appended to the dividend	10
Take	The number of decimals in the divisor	5
Remainder	The number of decimals in the quotient	5

PROOF.

Quotient = 66·79067—(A)
Divisor = 5·19878—(B)



From the dividend and ciphers appended = 347·2300000000
Take the remainder = 6174
347·2299993826—(C)

Ex. 4 Divide 87·3749 by ·00436.

```
·00436 ) 87·3749 ( 20040·1146
         872
         ---
           1749
           1744
           ----
              500
              436
              ---
               640
               436
               ---
               2040
               1744
               ----
                2960
                2616
                ----
                 344 Remainder.
```

From	The number of decimals and ciphers in the dividend	9
Take	The number of decimals in the divisor	5
Remainder	There remains the number of decimals that ought to be in the quotient	4

The proof would be similar to those previously given.

Ex. 5. Divide ·063478 by ·000125.

$$
\begin{array}{r}
\cdot 0000125\ )\ \cdot 063478\ (\ 5078\cdot 24 \\
625 \\
\hline
978 \\
875 \\
\hline
1030 \\
1000 \\
\hline
300 \\
250 \\
\hline
500 \\
500 \\
\hline
\end{array}
$$

Number of decimals and ciphers in the dividend = 9
Number in the divisor .... = 7
The number in the quotient .... = 2

16. ☞ To the previous remarks on division, we may add the following RULE.—Make the number of decimals in the divisor and dividend equal, by adding ciphers to the deficient one; then, when the divisor divides the dividend without annexing any more ciphers, the quotient will be a whole number; but when we call more ciphers to the dividend, any figures that are put after in the quotient will be decimals.

17. The reason that the number of decimals in the quotient is equal the number in the dividend with the ciphers appended, made less by the number in the divisor, will appear from the following; taking the 5th example for instance,

$$\text{Dividend} = \cdot 063478 = \frac{\cdot 063478}{1000000} = \frac{63478}{1000000}$$

$$\text{Divisor} = \cdot 0000125 = \frac{\cdot 0000125}{10000000} = \frac{125}{10000000}$$

$$\therefore\ \cdot 063478 \div \cdot 0000125 = \frac{63478}{1000000} \div \frac{125}{10000000} =$$

$$\frac{63478}{1000000} \times \frac{10000000}{125} = \frac{634780000000}{125000000} = \frac{634780}{125} =$$

$$5078\frac{30}{125} = 5078\frac{6}{25} = 5078\frac{24}{100} = 5078{\cdot}24.$$

Examples for Practice.

Ex. 6. Divide 354· by 3·1416. *Ans.* 112 681

Ex. 7. Divide 3·1416 by 89·74. *Ans.* ·0350078.

Ex. 8. Divide 2218·192 — ·7854 by 277·274 + 3·1416 × ·5236. *Ans.* 15·10236.

Those who have studied the history of the mathematical sciences cannot but have noticed the slow manner in which improvements have been admitted into general use; even at this late date, barbarous and inadequate as the method was, the Author was obliged to allow his bases to be expressed in vulgar fractions, in order that his published works on Dual Arithmetic might be better understood. Dual Arithmetic calls for but few innovations to establish its general notation; decimal arithmetic is left in possession of the full point (·), while in dual arithmetic the comma (,) is employed, a distinction easily remembered. The first notice of decimals is to be found in a tract at the end of Stevinus' *Arithmétique*, in the collection of his works by his friend and pupil Albert Girard; the tract is entitled *La Disme*. This collection was first published in Flemish, about the year 1590. At this early date, decimals in the first place are termed primes and marked (1), those in the second place are marked (2), and called seconds, and so on; whilst all integers are characterized by the sign (0), which is put after or above the last digit.

Example in Addition.

1590.	1865.
(0)(1)(2)(3)(4)	
3 4 6 1 2	3·4612
21 4 7 7 2	21·3472
1 3 0 0 6	1·3006
24 0 0 4 9	24·0049
50 1 1 3 9	50·1139
(0)(1)(2)(3)(4)	

The denominators 10; 100; 1000; &c., were employed after the time of Briggs and Napier. From what is here

shown it is presumed that the rationale of contracted processes and other decimal operations will be easily understood by the student.

The Metric System of Measurement.

For Multiples Greek words are used.

	Metres.	Feet.	Inches.
Metre	1·	3·2808992	39·3707904
Hecto-Metre	10·	32·808992	393·707904
Kilo-Metre	100·	328·08992	3937·07904
Kyria-Metre	1000·	3280·8992	39370·7904

For Divisors Latin words are used.

	Metres.	Feet.	Inches.
Metre	1·	3·2808992	39·3707904
Deci-Metre	·1	·32808992	3·93707904
Centi-Metre	·01	·032808992	·393707904
Milli-Metre	·001	·0032808992	·0393707904

18. Thus a Kilometre = 100 metres; and a Millimetre = a metre ÷ 1000.

The square Decametre, called the *Are*, is the element of land measure in France, and is equal 1076·42996 square feet English.

The Stere is a cubic metre = 35·316582 cubic feet English.

The Litre, for liquid measure, is a cubic decimetre = 1·76007 imperial pints, English, at the temperature of melting ice; a litre of distilled water weighs 154·34 *grains Troy.*

The unit of weight is the gramme; it is the weight of a cubic centimetre of distilled water, or of a millimetre, and hence equal to 15·434 *grains Troy.*

The English standard yard was destroyed by fire, copies of which are now employed that cannot be proved to be right or wrong, since the exact distance between the point of suspension and centre of oscillation, in any known pendulum, cannot be exactly measured.

Observations.

By removing the decimal point a figure to the left, a number is divided by 10; by moving the point two figures to the left, the number is divided by 100; and so on.

Thus, 3·937079 is the one-tenth of 39·37079,
·3937079 is the $\frac{1}{100}$ part of 39·37079,
·03937079 is the $\frac{1}{1000}$ part of 39·37079,
&c. &c.

Again, Multiply	39·37	39·37	39·37
by	·1	·01	·001
Product	3·937	·3937	·03937

39·3707904 × 10 = 393·707904
39·3707904 × 100 = 3937·07904
39·3707904 × 1000 = 39370·7904
&c. &c.

By moving the decimal point a figure to the right at each successive step.

·393707904 ÷ ·1 = 3·93707904
·393707904 ÷ ·01 = 39·3707904
·393707904 ÷ ·001 = 393·707904
·393707904 ÷ ·0001 = 3937·07904
&c. &c.

The same operations apply to other numbers and decimals.

EXAMPLES.

Ex. 1. Multiply 23·4 by 101· and also by 1·01.

```
 23·4          23·4
 101·          1·01
 ----          ----
 234            234
2340           2340
-----          -----
2363·4         23·634
```

OTHERWISE,

```
23·4    once
 ·234 one-hundredth part
------
23·634

2340·  100 times
  23·4 once
-------
2363·4
```

It is easily observed that the results are composed of the same figures, the positions of the decimal points being the only difference.

OR THUS,

00	23	40
23	40	
23 63 40		

23	40	00
	23	40
23 63 40		

19. The position of the decimal point is readily found, for 23·4 and 1·01 involve three decimals, then the result will be 23·634; ·234 × ·101 = ·023634; 2·34 × 1·01 = 2·3634; and so on.

Ex. 2. Multiply 12·34 by 1·1 twice in succession, the result by 1·01 three times in succession, and the last result by 1·001 four times in succession. Also multiply 1234· by 11· twice; 101· three times; and 1001· four times in succession continually.

12·34 once ⎫
1·234 one-tenth ⎭ 1·1
13·574
1·3574
14·9314 once ⎫
·149314 one-hundredth ⎭ 1·01
15·080714
·15080714
15·23152114
·1523152114
15·3838363514 once ⎫
·0153838363514 one-thousandth ⎭ 1·001
15·3992201877514
·0153992201877514
15·4146194079391514
·0154146194079391514
15·4300340273470905514
·0154300340273470905514
15·4454640613744376419514

SECOND PART OF THE EXAMPLE.

11· { once — 1234
ten times — 123 4
135·74
1357·4
101· { once — 1493·14
one hundred times — 149314·
150807·14

150807·14
15080714·

15231521·14
1523152114·

1001· { once 1538383635·14
1000 times 1538383635140·

1539922018775·14
1539922018775 40·

1541461940793915·14
1541461940793915140·

543003402734709055 14
1543003402734709055140·

1544546406137443764195·14

a result composed of the same figures as the former, the positions of the decimal point being the only difference.

Ex. 3. Multiply 47·35 by ·9 four consecutive times; ·99 three; and 999 twice continually together. And also find the value of

$$47·35 \times 9 \times 9 \times 9 \times 9 \times 99 \times 99 \times 99 \times 999 \times 999.$$

From 47·35 once } ·9
Take 4·735 $\frac{1}{10}$

42·615
4·2615

38·3535
3·83535

34·51815
3·451815

From 31·066325 once } ·99
Take ·31066325 $\frac{1}{100}$

30·75566175
·3075566175

30·448105 325
30448 051325

From 30·143624081175 once } ·999
Take ·030143624081175 $\frac{1}{1000}$

30·1134804570938 25
301134804570938 25

30·083266976636731175

SECOND PART OF THE EXAMPLE.

$$
\begin{array}{rll}
 & 473\cdot5 & \text{10 times} \\
 & \underline{47\cdot35} & \text{once} \\
47\cdot35 \times 9\cdot = & 4261\cdot5 & \text{10 times} \\
 & \underline{426\cdot15} & \text{once} \\
47\cdot35 \times 9\cdot \times 9\cdot = & 38353\cdot5 & \text{10 times} \\
 & \underline{3835\cdot35} & \text{once} \\
47\cdot35 \times 9\cdot \times 9\cdot \times 9\cdot = & 34518\cdot15 &
\end{array}
$$

20. Hence the figures composing the result stand as in the former case, but the decimal point assumes a different position.

Ex. 5. Find the value of $\cdot9 \times \cdot9 \times \cdot99 \times \cdot99 \times \cdot99 \times 101\cdot \times 11\cdot \times 11\cdot \times 11\cdot \times 2\cdot \times 1\cdot1 \times 1\cdot1 \times 1\cdot01 \times 1\cdot01 \times 1\cdot01 \times 1\cdot01$

$$
\begin{array}{rl}
101\cdot & \\
\underline{101} & \\
1111\cdot & \\
\underline{1111} & \\
12221 & \\
\underline{12221} & \\
134431\cdot & = 101\cdot \times 11\cdot \times 11\cdot \times 11\cdot \\
\underline{13443\cdot1} & \\
147874\cdot1 & \\
\underline{14787\cdot41} & \\
162661\cdot51 & \quad 1\cdot1 \times 1\cdot1 \\
\underline{1626\;6151} & \\
164288\cdot1251 & \\
\underline{1642\;881251} & \\
165931\cdot006351 & \\
\underline{1659\;31006351} & \\
167590\cdot31641451 & \\
\underline{1675\;9031641451} & \\
169266\cdot2195786551 & \quad 1\cdot01 \times 1\cdot01 \times 1\cdot01 \times 10\cdot1 \\
\underline{16926\;62195786551} & \\
152339\cdot59762078959 &
\end{array}
$$

152339 59762078959
 15233 959762078959

137105·637858710631 ·9 × ·9
 1371 05637858710631

135734·58148012352469
 1357 34581480123 52469

134377·23566532228944 31
 1343 772356653222894431

133032 46320866906654866 9 =

$(\cdot9)^2(\cdot99)^3(101\cdot)^1(11\cdot)^3(2)^1(1\cdot1)^2(1\cdot01)^4$, the signification of the indices 2, 3, 1, 3, 1, 2, 4, will be explained presently.

Ex. 6. Reduce the vulgar fraction $\frac{100000}{100001}$ to a decimal.

```
100001 ) 1000000 ( ·99999
          900009
          ------
           999910
           900009
           ------
            999010
            900009
            ------
             990010
             900009
             ------
                  1 Remainder.
```

Ex. 7. Reduce $\frac{100000000}{100000001}$ to a decimal.

Ans. ·99999999 and 1 Remainder.

Involution.

21. When a number is multiplied by itself the product is termed the square of that number, or its second power; the square of a number multiplied by itself is termed its cube, or third power; the cube of a number multiplied by itself is termed the fourth power, and so on. This process of multiplying a number a certain number of times into itself is called *Involution*, or raising of powers. The number continually multiplied by is called the *Root*, and the products are termed the *Powers*. If 2 be taken as a root, then

$$2=2=$$
$1+1$ the first power of 2;

$$2\times 2=4=$$
$1+2+1$ the 2nd power, or square of 2;

$$2\times 2\times 2=8=$$
$1+3+3+1$ the 3rd power, or cube of 2;

$$2\times 2\times 2\times 2=16=$$
$1+4+6+4+1$ the 4th power;

$$2\times 2\times 2\times 2\times 2=32=$$
$1+5+10+10+5+1$ the 5th power;

$$2\times 2\times 2\times 2\times 2\times 2=64=$$
$1+6+15+20+15+6+1$ the 6th power;

$$2\times 2\times 2\times 2\times 2\times 2\times 2=128=$$
$1+7+21+35+35+21+7+1$ the 7th power;

$$2\times 2\times 2\times 2\times 2\times 2\times 2\times 2=256=$$
$1+8+28+56+70+56+28+8+1$ the 8th power;

$$2\times 2\times 2\times 2\times 2\times 2\times 2\times 2\times 2=512=$$
$1+9+36+84+126+126+84+36+9+1$ the 9th.

22. *In dual arithmetic these numbers are of paramount importance, they are termed operative numbers.*

These results are readily found with the small number 2; but to raise any large number, as 235·78, to a high power involves much calculation and uncertainty by common arithmetic.

$$
\begin{array}{lll}
 & 1+1 \text{ Mult.} & =2; \\
 & \underline{1+1 \quad \text{by}} & =2; \\
 & 1+1 & \\
 & \underline{\quad 1+1} & \\
\text{Mult.} & 1+2+1 \text{ squ.} & =4; \\
\text{by} & \underline{1+1} & \\
 & 1+2+1 & \\
 & \underline{\quad +1+2+1} & \\
\text{Mult.} & 1+3+3+1 \text{ cube.} & =8; \\
\text{by} & \underline{1+1} & \\
 & 1+3+3+1 & \\
 & \underline{\quad +1+3+3+1} & \\
 & 1+4+6+4+1 \text{ fourth power} & =16; \\
 & \quad \&c. &
\end{array}
$$

		By decimals.	
Mult.	1+·1	1·1	
by	1+·1	·11	
	1+·1	1·21	=2nd.
	+·1+·01	·121	
Mult.	1+·2+·01	1·331	=3rd.
by	1+·1	·1331	
	1+·2+·01	1·4641	=4th.
	+·1+·02+·001	·14641	
Mult.	1+·3+·03+·001	1·61051	=5th.
by	1+·1	&c.	
	1+·3+·03+·001		
	+·1+·03+·003+·0001		
Mult.	1+·4+·06+·004+·0001		
by	1+·1		
	1+·4+·06+·004+·0001		
	+·1+·04+·006+·0004+·00001		
	1+·5+·1+·01+·0005+·00001		
	&c.		

These operations are given at great length, but when the numbers 1+1; 1+2+1; 1+3+3+3, &c., are known, any power of (1·1); (1·01); (1·001), &c., may be immediately found, as the succeeding examples will show.

Ex. 1. Find the 8th power of 1·1. The operative numbers in this case are 1; 8; 28; 56; 70; &c. (*See* page 19.)

1 8 8 6 0 6 8 81
2 5 7 5 2
2·1 4 3 5 8 8 81=$(1·1)^8$

The numbers 28; 56, &c., are set down diagonally.

Ex. 2. Required the 11th power of 1·1.

The requisite numbers are taken from the following table.

1 1 5 5 0 2 2 0 5 5 1 1
1 5 6 3 6 6 3 6 5 1
1 3 4 4 3 1
2·8 5 3 1 1 6 7 0 6 1 1 = $(1·1)^{11}$.

23. The following table, referred to in the last example, is easily formed and will be found useful.

TABLE.

A 1	E 1	1	1	1	1	1	1	1	1	1	1	1	B
D 1	G 2	3	4	5	6	7	8	9	10	11	12		
F 1	H 3	6	10	15	21	28	36	45	55	66			
1	4	10	20	35	56	84	120	165	220				
1	5	15	35	70	126	210	330	495					
1	6	21	56	126	252	462	792						
1	7	28	84	210	462	924							
1	8	36	120	330	792								
1	9	45	165	495									
1	10	55	220										
1	11	66											
1	12												
1													
C													

Fill the horizontal line of squares AB, and the vertical one AC with units, then the other numbers are found by adding the preceding number to that in the next square above it to the right in a diagonal direction. Thus 1 in the square D, + 1 in E = 2, in G; 1 in F + 2 in G = 3, in H, and so on. 210 + 252 = 462.

This table may be enlarged at pleasure. The numbers employed to find any power of 1·1, 1·01, 1·001, &c., or of 11·, 101·, 1001·, &c., are found on the diagonal lines. Thus $(1\cdot1)^{12}$ may be found as follows:—

1 2 6 0 5 2 4 2 5 0 6 2 1
1 6 2 9 9 2 9 9 2 6 1
2 4 7 9 7 4 2

$(1\cdot1)^{12} = 3\cdot1\ 3\ 8\ 4\ 2\ 8\ 3\ 7\ 6\ 7\ 2\ 1$

Mathematicians have found it convenient to represent any number, as 1·1, raised to any power, as 12, thus, $(1\cdot1)^{12}$; this notation will be fully explained presently.

Ex. 3. Find the 3rd power of 1·001.

Ans. 1·003003001.

Ex. 4. Find the 7th power 1·01.

Ans. 1·0721353521701.

24. For the sake of brevity the power is expressed by a small figure written a little above the root. Thus, 8^4 is the notation employed to denote $8 \times 8 \times 8 \times 8 = 4096$. 4 in this case is called the *index* or *exponent*, 8 the *root*, and 4096 the *power*.

☞ It should be observed that conventional arrangements may indicate processes precisely, and yet render but little or no assistance to an operator trying to obtain results. This is a grave objection which applies to many of our modern mathematical researches and formulæ.

When two or more powers of the same number are multiplied together, the index of the product is the sum of the indices of the factors to be multiplied.

EXAMPLES.

Ex. 1. $7^5 \times 7^3 = 7^8$, for
$(7\times7\times7\times7\times7)\times(7\times7\times7)=$
$7\times7\times7\times7\times7\times7\times7\times7=7^8$,
or $5+3=8$.

Ex. 2. $2\times2\times2\times2\times2=2^5\times2^1=2^6$, or $5+1=6$.

Ex. 3. $6^3\times6^5\times6=6^9$.

6 is counted 6^1, since unity is supposed to be the index when none is expressed.

When one power is divided by another of the same number, the index of the quotient is found by subtracting the index of the divisor from that of the dividend.

$$\text{Thus } 9^{11}\div9^4=\frac{9\times9\times9\times9\times9\times9\times9\times9\times9\times9\times9}{9\times9\times9\times9}=9^7.$$

$$\frac{(22)^5}{22}=(22)^4,\text{ for }\frac{22\times22\times22\times22\times22}{22}=22\times22\times22\times22.$$

It is readily shown that when the index is reduced to zero the result $=1$. Thus

$1=\frac{3^5}{3^5}=3^\circ$, the same may be said of 9°; 10°; 11°, &c., in continuing the proposed system of notation, and further $5^6\div5^8=5^{-2}=\frac{5^6}{5^8}=\frac{1}{5^2}$. Hence

$\frac{1}{5^2}$ may be written 5^{-2}

$\frac{1}{3^3}$ may be written 3^{-3}

and so on.

It is easily observed, that to represent a power raised to another power, the index of the given power must be multiplied by the index of the required one. For 6^3 raised to the fourth power may be represented by 6^{12}; as

$$6^3 \times 6^3 \times 6^3 \times 6^3 = 6^{3+3+3+3} = 6^{12}.$$

In the same way 3^2 raised to the 5th power may be represented be $3^{2\times5}=3^{10}$. The same rule may be applied in other cases.

THE FIRST NINE POWERS OF THE FIRST NINE NUMBERS.

1st	2nd	3rd	4th	5th	6th	7th	8th	9th
1	1	1	1	1	1	1	1	1
2	4	8	16	32	64	128	256	512
3	9	27	81	243	729	2187	6561	19683
4	16	64	256	1024	4096	16384	65536	262144
5	25	125	625	3125	15625	78125	390625	1953125
6	36	216	1296	7776	46656	279936	1679616	10077696
7	49	343	2401	16807	117649	823543	5764801	40353607
8	64	512	4096	32768	262144	2097152	16777216	134217728
9	81	729	6561	59049	531441	4782969	43046721	387420489

EVOLUTION.

Evolution is the reverse of Involution, its object being to extract or find the roots of given powers. Thus, the second or square root of 49 is 7, because $7\times7=49$; the fifth root of 7776 is 6, because $6\times6\times6\times6\times6=7776$.

The few roots that can be found exactly are called *rational roots*. Although the roots of many numbers cannot be found exactly, Dual Arithmetic shows how to find them by a direct and simple calculation to as great a degree of accuracy as we please. Roots which cannot be found exactly are termed *irrational roots* or *surds*. The square root of 2 is a surd, since no number multiplied by itself will exactly produce 2; the square root of 81 is rational, it being exactly equal 9. There are many plans given to express roots without any prescribed method of performing the operations indicated. Roots are often denoted by placing the mark ($\sqrt{\ }$) before the power, with the index of the root prefixed. In this way the fifth root of 2 is expressed by $\sqrt[5]{2}$, and the square root of 8 by $\sqrt{8}$, in denoting the square root by this devise the index 2 is generally omitted. Roots are often indicated like powers, but with *fractions as indices*; thus, the square root of 11 is

written $11^{\frac{1}{2}}$; the cube root of 5 is written $5^{\frac{1}{3}}$; the 5th root of 2 is written $2^{\frac{1}{5}}$, &c. There is an analogy in this extension. For, if any number, 8^2, be raised to the 3rd power, then $8^{2\times 3}=8^6$.

Therefore conversely, the third root of 8^6 is 8^2, obtained by dividing 6 by 3.

Hence 6 may be considered as the index of the power, and 3 that of the root, and $8^{\frac{6}{3}}$ may be either the sixth power of the cube root of 8, or the cube root of the 6th power of 8, since the third root of $8\times 8\times 8\times 8\times 8\times 8=8^2$; and the 6th power of $\sqrt[3]{8}$, or

$$\sqrt[3]{8}\times\sqrt[3]{8}\times\sqrt[3]{8}\times\sqrt[3]{8}\times\sqrt[3]{8}\times\sqrt[3]{8}=8\times 8=8^2.$$

Hence, when the indices are integers in the form of fractions, the denominator signifies the index of the root, and the numerator the index of the power. When a number has a fractional index, the numerator shows the power to which the number has to be raised, and the denominator the root to be extracted. Thus,

$$7^{\frac{2}{3}}=\sqrt[3]{7^2},\text{ for}$$

$$\sqrt[3]{7^2}\times\sqrt[3]{7^2}\times\sqrt[3]{7^2}=7^2=7^{\frac{2}{3}+\frac{2}{3}+\frac{2}{3}}.$$

Now, an even root of a negative number cannot be extracted, but an odd root can. Thus, the cube root of -64, or $(-64)^{\frac{1}{3}}=-4$, for

$$(-4)\times(-4)\times(-4)=-64.$$

The square root of -64 written $(-64)^{\frac{1}{2}}$ is not $=+8$, nor is it equal -8, for $(+8)\times(+8)=+64$; and also $(-8)\times(-8)=+64$. Although the square root of -64 cannot be taken, yet $\frac{50000}{100001}$ root can be extracted of a negative number, because 100001 is odd, and $\frac{50000}{100001}$ very nearly $=\frac{1}{2}$. By dual arithmetic the value of $(-64)^{\frac{50000}{100001}}$ is readily found without the use of tables. The values of $(-3)^{\frac{100000000}{100000001}}$, of $(-1)^{\frac{1000000000}{1000000001}}$, &c., may be found also, so we may approach an even root of a negative number; now for the first time thus indicated.

Examples for Practice.

1. Find the value of $(-1)^{\frac{50}{101}}$. *Ans.* $+1$.
   For $(-1)^{50}=+1$ and $(+1)^{\frac{1}{101}}=+1$
   or $(-1)^{\frac{1}{101}}=-1$ and $(-1)^{50}=+1$
2. Find the value of $(-1)^{\frac{999}{1001}}$. *Ans.* -1.
   For $(-1)^{999}=-1$ and $(-1)^{1001}=-1$.
3. Find the value of $(-1)^{\frac{499}{1001}}$. *Ans.* -1.

DUAL ARITHMETIC,

A NEW ART.

CHAPTER I.

DEFINITIONS, SIGNS, AND NOTATION.

It is not presumed that the student will understand the processes indicated in this chapter, or remember the symbols of operation, until he can perform the operations indicated and has acquired some practical experience.

25. Dual Arithmetic is *a new art of manœuvering numbers* and investigating the relations of quantities with ease and accuracy, with or without the use of tables.

26. The term *Dual* is employed because the art has *two branches*, the basis of each branch being composed of *two parts*, and because the digits of a dual number may be subjected to a variety of changes in magnitude and position, while at the same time remaining equal in value to *two unchangeable extremes*, namely, a natural number and a logarithm to a known base.

27. Since the digits of a dual number are susceptible of a vast variety of changes without altering its two ultimate values, dual numbers may be said to be changeable without being variable.

28. In Dual Arithmetic the Arabic figures 1, 2, 3, 4, 5, 6, 7, 8, 9 and 0 are employed.

(0) is called naught, zero, or a cipher, when alone. (0) represents the beginning of numbers positive and negative, and the beginning of things.

Arabic figures and notation have not been known in Europe more than 900 years, and but little used until after 1600 A.D. Decimal arithmetic, as now taught in schools, is not more than 120 years in use. Leonardo Bonacci, a merchant of Pisa, introduced the Arabian system of Digital Arithmetic into Italy, and wrote the first treatise, published in Europe, about the year 1228 A.D.

A number *larger* than any that may be named is ex-

pressed by the symbol ∞ with the sign of *plus*, as $+\infty$. A number smaller than any that may be named is expressed by combining the *minus* sign ($-$) with the character ∞, as $-\infty$.

$+\infty$, greater than any number that may be named.

$-\infty$, less than any number that may be named.

$+\infty$ is sometimes read *plus infinity;* and $-\infty$, *minus infinity*.

Numbers in the dual system of arithmetic are expressed by the continued product of the powers of one or more of the following bases which are seldom introduced into the figurate operations of the art.

Bases of the Ascending Branch.

29. The bases of this branch of the art can be expressed as follows:—

$+\infty$ ..... $(10000+1)$; $(1000+1)$; $(100+1)$; $(10+1)$; $(1+1)$; $(\frac{1}{10}+1)$; $(\frac{1}{100}+1)$; $(\frac{1}{1000}+1)$; 1;

more conveniently written thus,

$+\infty$ .... 10001; 1001; 101; 11; 2; 1·1; 1·01; 1·001; .... 1 the limit,

increasing in magnitude from right to left.

These bases are less and less as they approach 1, but cannot be less than 1.

Bases of the Descending Branch.

30. These bases may be expressed thus,

$-(\infty)$ .... $(1-1000)$; $(1-100)$; $(1-10)$; $(1-1)$; $(1-\frac{1}{10})$; $(1-\frac{1}{100})$; $(1-\frac{1}{1000})$; ...(1),

but more concisely written,

$-\infty$ .... -999; -99; -9; 0; ·9; ·99; ·999; 1,

decreasing in magnitude from right to left.

This scale of bases approaches 1, but cannot be greater than 1.

Bases of Common and Decimal Arithmetic.

31. In this work the numbers of common and decimal arithmetic are sometimes termed ordinary, common, or natural numbers.

$+\infty$ .... 1000·; 100·; 10·; 1·; ·1; ·01; ·001; .... 0, approaching 0, but cannot be equal to or less than 0, or zero.

Some examples will make clear anything that may seem too abstract in the preceding generalities.

The diameter of the earth through the poles is said to be

equal 7898·8809 statute miles, of 5280 feet each; therefore the diameter $= 41706091{\cdot}152$ feet, which, according to usage, is a contracted method of expressing

$$4(10000000\cdot) + 1(1000000\cdot) + 7(100000\cdot) + 6(1000\cdot) + 9(10\cdot) + 1 + \tfrac{1}{10} + \tfrac{5}{100} + \tfrac{2}{1000};$$

which, according to the method agreed upon to express powers, becomes

$$4(10)^7 + (10)^6 + 7(10)^5 + 6(10)^3 + 9(10)^1 + 1 + (10)^{-1} + 5(10)^{-2} + 2(10)^{-3}.$$

In common arithmetic 4, 1, 7, &c. are termed digits, and do not exceed 9.

32. In dual arithmetic the powers of the dual *bases* are only registered. Thus 41706091·152 is equal to

$$(\cdot 99999)^1\ (\cdot 999999)^3\ (\cdot 9999999)^3\ (\cdot 99999999)^6\ (1+1)^2\ (1\cdot 01)^4\ (1\cdot 001)^2,$$

when multiplied by 10^7. The bases being omitted, this dual number is written

$$41706091{\cdot}152$$

$$\|$$

$$'0'0'0'0'1'3'3'6 \uparrow 10^7 2^2 \downarrow 0,4,2,0,0,0,0,0, \quad \text{(A)}.$$

$$=$$

$$10^7 2^2 \downarrow 0,4,1,9,8,6,9,6, \quad \text{(B)}.$$

$$=$$

$$'8'3'1'4'6'8'1'0 \uparrow 10^8 \quad \text{(C)}.$$

$$=$$

$$'0'0'1'5'7'0'8'4 \uparrow 10^7 \downarrow 15,0,0,0,0,0,0,0, \quad \text{(D)}.$$

&c.

The student is not expected to know how these dual numbers are obtained until he understands the methods of reduction explained and exemplified in Chapters II., III., and IV.

Referring to the extended form (A), '3; 4, 2, &c., are called dual digits, and express the powers of the bases involved, and, unlike the digits of ordinary arithmetic, may be greater or less than 9. The zero between $\downarrow$ and the first 4, in (A) shows that no power of 1·1 is employed, while the ciphers after 2 show that the bases 1·0001; 1·00001, &c., are not involved. The position of

a dual digit before, between, or after the signs ↑ ↓ points out its value. These arrangements will be discussed hereafter.

$$\underset{p}{{}_{n}{\uparrow}_{m}} \qquad \overset{p}{{}_{m}{\downarrow}_{n}}$$

A small figure placed at p designates the position occupied by a dual digit, and sometimes points out the leading position occupied by the first of more dual digits than one.

m	expresses	10^m
$\overline{m}$	"	$\frac{1}{10^m}$
n	"	2^n
$\overline{n}$	"	$\frac{1}{2^n}$

$$\overset{p}{\underset{p}{{}_{m}{\updownarrow}_{n}}}$$

33. ☞ The comma (,) is employed in the operations of dual arithmetic, while the period (.) is retained to separate whole numbers from decimal fractions. This part of the general notation should be remembered, (17), page 12.

Articles are referred to thus (17), refers to article 17.

34. A dual number of positive dual digits has always an exact value in common numbers when no contractions are employed in the reduction.

When eight positions to the right and eight to the left of the signs ↑ ↓, counting from left to right in both cases, are occupied by ciphers or other digits, the sign ↓ being placed before the eight ascending digits and ↑ after the eight descending; yet with respect to range, the dual number is said to be one of eight digits, although sixteen positions, and other positions between the signs ↑ and ↓, may be occupied.

If one of the signs ↑ or ↓ is omitted, the positions attached to the other are supposed to be occupied by ciphers.

35. ☞ When the last dual digit and all that follow are rejected, and when the last digit is 5, 6, 7, 8, or 9, the digit preceding may be counted one more, as in decimal arithmetic, (6).

36. For most practical purposes common arithmetical results are required true, to not more than seven places of figures. To obtain this degree of accuracy, eight consecutive dual digits must be employed. In making calculations

the allowances specified (6), (35), must be attended to. A little additional calculation will secure results true to 8. 9. 10. . . . places of figures. Common logarithms to seven places of decimals do not secure as great a degree of accuracy, and cannot be independently tested and extended.

37. Nine dual digits give results true to eight places of figures.

Ten dual digits give results true to nine places of figures.

Eleven dual digits give results true to ten places of figures.

And so on.

Results obtained by the use of tables of seven-place logarithms cannot be true to seven places of decimals, but may be true to six places of figures, counting whole numbers and decimals; this fact is seldom stated.

38. A dual number is easily transformed into another, all of whose digits being reduced to ciphers, except the last. The transformation of a dual number of eight digits into another, whose first seven digits are ciphers, is termed *reducing a dual number to the eighth position.* A dual number reduced to the eighth position is called a dual logarithm.

$$2\cdot = \downarrow 7,2,6,0,7,8,2,6, = \downarrow 0,0,0,0,0,0,0,69314718,$$

$$= \overset{8}{\downarrow} 69314718,$$

In practice the 8 is omitted, and the expression is written

$2\cdot = \downarrow 69314718$, which represents $(1\cdot00000001)^{69314718}$.

Then 69314718, is termed the dual logarithm of 2· written

$$\downarrow, (2\cdot) = 69314718,$$

The dual logarithm of 41706091·152 is equal to the whole number 1754615775, or

$$\downarrow, (41706091\cdot152) = 1754615775,$$

The dual logarithms of common numbers are easily found, as well as the common numbers corresponding to dual logarithms, without the use of tables.

NOTATION. ASCENDING BRANCH.

39. The notation, although new, is easily remembered from its symmetry, compactness, and uniformity.

1 is represented by $\downarrow$

ONE DECIMAL.			FIRST POSITION.
1·1	„	„	$\downarrow 1$,
$(1\cdot1)^2$	„	„	$\downarrow 2$,
$(1\cdot1)^3$	„	„	$\downarrow 3$,
&c.		&c.	

TWO DECIMALS. SECOND POSITION.

$(1\cdot01)$ is represented by $\downarrow 0,\ 1$, or $\downarrow^{2} 1$,
$(1\cdot01)^2$ „ „ $\downarrow 0,\ 2$, or $\downarrow^{2} 2$,
$(1\cdot01)^3$ „ „ $\downarrow 0,\ 3$, or $\downarrow^{2} 3$,
&c. &c.

THREE DECIMALS. THIRD POSITION.

$(1\cdot001)$ is represented by $\downarrow 0,\ 0,\ 1$, or $\downarrow^{3} 1$,
$(1\cdot001)^2$ „ „ $\downarrow 0,\ 0,\ 2$, or $\downarrow^{3} 2$,
$(1\cdot001)^3$ „ „ $\downarrow 0,\ 0,\ 3$, or $\downarrow^{3} 3$,
&c. &c.

$(1\cdot1)^5(1\cdot01)$ is represented by $\downarrow 5,\ 1$,
$(1\cdot1)^7(1\cdot01)^2$ „ „ $\downarrow 7,\ 2$,
$(1\cdot1)^3(1\cdot01)^4(1\cdot001)^5$ „ „ $\downarrow 3,\ 4,\ 5$,

$(1\cdot001)^6(1\cdot00001)^2(1\cdot00000001)^8$ is expressed by
$\downarrow 0,0,6,0,2,0,0,8$,

$\downarrow 0,0$, in the first and second positions, indicates that no power of $(1\cdot1)$ or of $(1\cdot01)$ is involved. The cipher in the fourth position indicates that no power of $1\cdot0001$ is involved; the same may be said of other positions.

$(1+1)$ is represented by $(2)\downarrow$ or $\downarrow^{1}$
$(1+1)^2$ „ „ $(4)\downarrow$ or $\downarrow^{2}$
$(1+1)^3$ „ „ $(8)\downarrow$ or $\downarrow^{3}$
$(1+1)^{-3}$ „ „ $(\frac{1}{8})\downarrow$ or $\downarrow^{\bar{3}}$
&c. &c.

1	2 8	7th power of 2
1	6	4th „ „ 2
2		1st „ „ 2
2	5 6	8th „ „ 2
3	2	5th „ „ 2
4		2nd „ „ 2
5	1 2	9th „ „ 2
6	4	6th „ „ 2
8		3rd „ „ 2

When decimal points are introduced, the numbers $1\cdot28$; $1\cdot6$; $2\cdot$; $2\cdot56$; &c. range in order between 1 and 10.

The continued product $(1\cdot1)^3(1\cdot01)^2(1\cdot001)^5(11\cdot)(101)^2$ may be written $2,1,\downarrow 3,2,5$, no power of $1+1=2$ being employed.

Because $(11)(101)^2=(10)(100)^2(1\cdot1)(1\cdot01)^2$
$\therefore\quad 2,1,\downarrow 3,2,5,=(10)\,{}^5\!\downarrow 4,4,5,={}^5\!\downarrow 4,4,5,$

Hence the dual digits to the left of $\downarrow$ can always be transferred to the right of $\downarrow$.

$\downarrow 0,0,6,0,2,0,0,5,$ may be written $\overset{3}{\downarrow}\, 6,0,2,0,0,5,$ or thus, $\overset{3}{\downarrow} 6, \overset{5}{\downarrow} 2, \overset{8}{\downarrow} 5,$

Again,

(11) is represented by $1,\downarrow$
$(11)^2$ „ „ $2,\downarrow$
$(11)^3$ „ „ $3,\downarrow$
&c. &c.

(101) is represented by $1,0,\downarrow$ or $1,\overset{2}{\downarrow}$
$(101)^2$ „ „ $2,0,\downarrow$ or $2,\overset{2}{\downarrow}$
$(101)^2$ „ „ $3,0,\downarrow$ or $3,\overset{2}{\downarrow}$
&c. &c.

$(1001)^6$ is expressed by $6,0,0,\downarrow$ or $6,\overset{3}{\downarrow}$

$(1+1)^3(11\cdot)^4(101\cdot)^3=3,4,(8)\downarrow$ which is reduced to $(2)^3(10)^{10}\downarrow 4,3,$ or ${}^{10}\!\downarrow 4,3,$ without mental labour. See Reduction of Dual Arithmetic.

NOTATION. DESCENDING BRANCH.

40. In this branch the arrow points *up*, and the comma is to the left of the digit and *above*, while in the ascending branch the arrow points *down*, and the comma is to the right of the digit and *below*.

1 is represented by $\uparrow$
One decimal $(\cdot9)$ „ „ $'1\uparrow$ in the first position.
„ „ $(\cdot9)^2$ „ „ $'2\uparrow$ „ „ „
„ „ $(\cdot9)^3$ „ „ $'3\uparrow$ „ „ „
&c. &c.

TWO DECIMALS. SECOND POSITION.

$(\cdot99)$ is represented by $'0'1\uparrow$ or $'1\,{}_2\!\uparrow$
$(\cdot99)^2$ „ „ $'0'2\uparrow$ or $'2\,{}_2\!\uparrow$
$(\cdot99)^3$ „ „ $'0'3\uparrow$ or $'3\,{}_2\!\uparrow$
$(\cdot99)^4$ „ „ $'0'4\uparrow$ or $'4\,{}_2\!\uparrow$
&c. &c.

THREE DECIMALS. THIRD POSITION.

$(\cdot999)$ is represented by $'0'0'1\uparrow$ or $'1\,{}_3\!\uparrow$
$(\cdot999)^2$ „ „ $'0'0'2\uparrow$ or $'2\,{}_3\!\uparrow$
$(\cdot999)^3$ „ „ $'0'0'3\uparrow$ or $'3\,{}_3\!\uparrow$
&c. &c.

41. ☞ In both branches, if there be n decimals in any base, its powers or dual digits are placed in the nth position. ☜

$$(\cdot 9)^3(\cdot 99)^2 \text{ is written } '3'2\uparrow$$
$$(\cdot 9)^7(\cdot 99)^5 \quad ,, \quad '7'5\uparrow$$
$$(\cdot 999)^3(\cdot 999999)^2(\cdot 99999999)^6 \text{ is written } 'o'o'3'o'o'2'o'6\uparrow$$

42. A cipher being in the first and also in the second position, shows that no power of ·9 or ·99 is employed; the same may be said of other positions occupied by ciphers.

$$'o'o'3'o'o'2'o'6\uparrow \text{ may be written } '3'o'o'2'o'6\,{}_3\!\uparrow =$$
$$'3\,{}_3\!\uparrow '2\,{}_6\!\uparrow '6\,{}_8\!\uparrow$$

43. In the descending branch, except in analytical inquiries, the base $(1-1)=0$ is omitted. The introduction of a dual digit with this base would cause the whole dual number, with which it is united, to vanish.

44. Again, $(9\cdot)$ is represented by $\uparrow '1$
$(9.)^2$,, ,, $\uparrow '2$
$(9\cdot)^3$,, ,, $\uparrow '3$
&c. &c.

$(99\cdot)$ is represented by $\uparrow 'o'1$ or $\uparrow_2 '1$
$(99\cdot)^2$,, ,, $\uparrow 'o'2$ or $\uparrow_2 '2$
$(99\cdot)^3$,, ,, $\uparrow 'o'3$ or $\uparrow_2 '3$
&c. &c.

$$(\cdot 9)^3(\cdot 99)^2(\cdot 999)^5(9\cdot)(99\cdot)^2 \text{ is written } '3'2'5\uparrow '1'2.$$

and may be put under the form $'4'4'5\uparrow(10)^5='4'4'5\uparrow$.

The bases 9· 99· 999· &c. are seldom introduced, unless when the descending dual branch is employed apart from the ascending branch. These bases are not included in the general scales of bases. The reduction of $'3'2'5\uparrow '1'2$ to $'4'4'5\uparrow(10)^5$ is similar to that established for the ascending branch.

For $(1\cdot 1)^3(1\cdot 01)^2(1\cdot 001)^5(11\cdot)(101)^2 = 2,1,\downarrow 2,5, =$
$^5\!\downarrow 4,4,5,=(10)\,{}^5\!\downarrow 4,4,5,$ (39).

45. Dual digits may be vastly greater than 9; for example:—

$$(10)^5(2)^3(\cdot 99999999)^{57893262}(1\cdot 00000001)^{87654321}$$

is expressed thus, $\quad '57893262\,{}_8\!\uparrow(10)^5\,(2)^3\downarrow^8 87654321,$

or thus, $\quad '57893262\ {}_5\!\overset{8}{\underset{8}{\updownarrow}}{}_3\ 87654321,$

the power of (10) being marked to the left, and the power of (2) to the right at the middle of the double arrow. The positions are indicated above to the right and below to the left.

46. Any ordinary number may be expressed by a dual number, each of whose digits is not greater than 9, by employing but one branch. But by a skilful use of both branches of the art combined, any common or natural number may be represented by dual digits not greater than 5. For example 1·03 is equal

$$\text{'0'0'0'3'0'2'2'0'0'2'5'7'9'5} \updownarrow \text{0,3,0,0,1,0,5,5,} = 1{\cdot}03$$

$$\|$$

$$1{\cdot}03 = \text{'3'0'2'2'0'0'2'5'7'9'5} \updownarrow^{2}_{4} \text{3 0,0,1,0,5,5,}$$

$$\|$$

$$\text{'3} {\uparrow}_{4} \text{'2'2} {\uparrow}_{6} \text{'2'5'7'9'5} {\uparrow}_{10} {\downarrow}^{2} \text{3,} {\downarrow}^{5} \text{1,} {\downarrow}^{7} \text{5,5,} = 1{\cdot}03$$

The digits 7 and 9 of this example, in the 12th and 13th positions, on the descending side, are not reduced below 6, as it was more convenient to have them greater.

47. In the descending branch, as in the ascending, a dual number reduced to the eighth position is also called a dual logarithm, and must be considered negative, if the descending dual logarithm is taken positive, and *vice versâ.*

It will be shown hereafter, that

$$\text{'10536052} {\uparrow}_{8} = \text{'1} \uparrow$$

$$\text{'1005034} {\uparrow}_{8} = \text{'0'1} \uparrow$$

$$\text{'100050} {\uparrow}_{8} = \text{'0'0'1} \uparrow$$

$$\text{'10000} {\uparrow}_{8} = \text{'0'0'0'1} \uparrow$$

&c. &c.

Then $\quad \text{'2'3'4'5'6'7'8'9} \uparrow = \text{'24544195} {\uparrow}_{8}$

For twice '10536052=	'21072104
3 times '1005034=	3015102
4 times '100050=	400200
And	56789
	'24544195 $\uparrow_8$

The 8 designating the position is omitted in practice (38).

Again,

$$\cdot 765432110 = \text{'3'0'1} \updownarrow \text{0,5,0,0,1,5,6,3,}$$

It is readily shown that $\downarrow 0,5,0,0,1,5,6,3, = \overset{8}{\downarrow} 4976728,$
and that $'3'0'1 \uparrow = '31708206 \underset{8}{\uparrow}$

Then $\cdot 76543211 = '31708206 \updownarrow 4976728,$

$$\begin{array}{r} '31708206 \\ 4976728, \\ \hline '26731478 \end{array}$$

Therefore, the dual logarithm of the decimal $\cdot 76543211$ is $'27631478$ written $\downarrow$, $(\cdot 76543211) \doteq '26731478$
and $\therefore$ $\cdot 76543211 = '26731478 \underset{8}{\uparrow}$ (38).

These reductions are introduced to exemplify the notation; how to make them will be shown hereafter.

CHAPTER II.

DUAL ARITHMETICAL REDUCTIONS.

48. Here it may be necessary to observe that hitherto we have not entered upon dual developments, or established any of the leading principles of dual arithmetic, nor performed any practical operations with this art, but merely defined terms, described symbols of operation, sketched the general notation, and introduced such auxiliary matter as might tend to render what follows easily intelligible to those but slightly acquainted with decimal arithmetic.

TO REDUCE DUAL TO COMMON NUMBERS WITHOUT THE USE OF TABLES.

49. The operative numbers, or coefficients, tabulated (23), may be determined by different operations, some of which are explained at the end of the work. These numbers, of so much importance in dual arithmetic, may be quickly determined, and at the same time arranged in a triangular form; for let units be placed in cells on the sides AB, AC, then the remaining cells of each succeeding vertical row between the dotted lines from A to CB are filled by continually adding each number to the one immediately following it on its right, between any two dotted lines, to obtain the next vertical row in succession, registering the results *in the cells not occupied* by units, beginning at A.

	A										
	↓										
For digit 1	1	1									1, or ’1 in any position.
For digit 2	1	2	1								2, or ’2 in any position.
For digit 3	1	3	3	1							3, or ’3 in any position.
&c.	1	4	6	4	1						&c.
	1	5	10	10	5	1					
	1	6	15	20	15	6	1				
	1	7	21	35	35	21	7	1			
	1	8	28	56	70	56	28	8	1		
C	1	9	36	84	126	126	84	36	9	1	B
			&c.			&c.					

These numbers, generally termed binomial coefficients, are employed to find the powers of the bases ·9; ·99; ·999; &c., as well as the powers of the bases 1·1; 1·01; 1·001; &c.

But it must be observed that the numbers in the second, fourth, sixth, &c. perpendicular columns are to be considered negative and subtracted in the descending branch. (19), (20), (22).

Problem I.

50. *To find the ordinary number answering to a single digit of either branch in any position.*

Rule.

The coefficients for the given digit are set down with no ciphers between them for the first position, one cipher for digits in the second position, and so on. When the coefficients consist of two or more figures, these figures must be arranged diagonally from right to left, falling into horizontal and vertical rows, the units on the first horizontal row, and then the whole summed with proper regard to negative numbers, if for digits of the descending branch.

EXAMPLES.

Ex. 1. Set down the common number answering to ↓ 9,

1 9/6/4/6/6/4/6/91

/3/8/2/2/8/3/

/ /1/1/ / /

↓ 9,= 2·3 5 7 9 4 7 6 91

Ex. 2. Set down the common number answering to ↓ 12, and also to ↓ 0,5, and ↓ 0,0,7,

1/2/6/0/5/2/4/2/5/0/6/2/1

/1/6/2/9/9/2/9/9/2/6/1/

/ /2/4/7/9/7/4/2/ / /

↓ 12,=3·1 3 8 4 2 8 3 7 6 7 2 1

1·05/0/0/0/0/0/501

/0/1/0/1/0/

↓ 0,5,= 1·05 1 0 1 0 0 501

1·00700/1/00/5/00/1/007001

/2/ /3/ /2/

↓ 0,0,7,= 1·00702 1 03 5 02 1 007001

These results may also be found by a series of continual additions, (18), (19), (20).

ANOTHER METHOD.

Ex. 3. Find the ordinary numbers that ↓ 3,; ↓ 12,; ↓ 0,5, and ↓ 0,0,7, represent, true to eight places of decimals.

↓ 3,=1·33100000 has merely to be set down.

	K	J	I	H	G	F	E	D	C	B	A		
			1	0	0	0	0	0	0	0	0	A	1
			1	2	0	0	0	0	0	0	0	B	12
				6	6	0	0	0	0	0	0	C	66
				2	2	0	0	0	0	0	0	D	220
					4	9	5	0	0	0	0	E	495
						7	9	2	0	0	0	F	792
							9	2	4	0	0	G	924
								7	9	2	0	H	792
									4	9	5	I	495
										2	2	J	220
												K	66
$\downarrow 12,$ =			3	·1	3	8	4	2	8	3	8		

☞ 51. Such examples as this are seldom required in practice; it is introduced to illustrate a principle.

The operating multipliers, or coefficients, are in a column to the right, each on the line it produces. The first horizontal row A is composed of a unit and eight ciphers determined by the range of accuracy required; this row is then divided into *periods of single figures*, because $\downarrow$ 12, is in the first position; it must be divided into *periods of two figures each* when the dual digit is in the second position; into *periods of three figures each* when the dual digit is in the third position, and so on from left to right, neglecting ciphers if there be any to the left of A.

The horizontal row B is found by multiplying A by 12 beginning at B; the horizontal row C is found by multiplying A by 66 beginning at C; the horizontal row D is found by multiplying A by 220 beginning at D, and so on to K, (21), (22), (18), (19), for which 1 is set down, since ·66 is rejected. The sum agrees with that given in *Ex.* 2, to the required degree of accuracy.

	E		D		C		B		A			Coefficients.
	1	0	0	0	0	0	0	0	0	·	A	1
			5	0	0	0	0	0	0	·	B	5
				1	0	0	0	0	0	·	C	10
$\downarrow^2 5,$						1	0	0	0	·	D	10
									5	·	E	5
$\downarrow 0.5,$ =	1	·0	5	1	0	1	0	0	5			

The last period is completed by affixing a dot.

	D	C			B			A				
		1	0	0	0	0	0	0	0	0	A	1
					7	0	0	0	0	0	B	7
$\downarrow 7,^{3}$							2	1	0	0	C	21
										4	D	35

$\uparrow 0,0,7, \;=\; 1\cdot 0\ 0\ 7\ 0\ 2\ 1\ 0\ 4$

D, in the first horizontal row A, contains no figures, yet in multiplying by 35 we have to carry 3·5 from C; hence (6) 4 is set down from multiplying the period D by 35.

52. *To reduce a single dual digit of the descending branch to an ordinary number.*

Ex. 4. Write down the common number answering to '3 ↑

The coefficients are 1; 3; 3; 1, every second one being made negative, these numbers may be written

$1\bar{3}3\bar{1}$

'3 ↑= ·729, the negative numbers being taken from 10.

Ex. 5. Reduce '0'7 ↑ or '7 ,↑ to a common number.

$1\ \ 0\ \ \bar{7}\ \ 0\ \ 1\ \ 0\ \ \bar{5}\ \ 0\ \ 5\ \ 0\ \ \bar{1}\ \ 0\ \ 7\ \ 0\ \ \bar{1}$

$2\ \ \ \bar{3}\ \ \ 3\ \ \ \bar{2}$

'7 ,↑ = ·9 3 2 0 6 5 3 4 7 9 0 6 9 9

52. The numbers 1; 7; 21; 35; 35; 21; 7; 1, with their signs changed as before directed (49), become

1;—7; 21;—35; 35;—21; 7;—1.

These numbers are set down diagonally from right to left, keeping the units on the upper line, placing single ciphers between them. No ciphers are introduced for the base ·9 or 9·; one cipher for the base ·99 or 99·; two ciphers for the base ·999 or 999·, and so on.

SECOND METHOD BY COMMON SUBTRACTION.

'0'1 ↑=	·99
	99
'0'2 ↑=	·9801
	9801
'0'3 ↑=	·970299

’o’3 ↑= ·970299
970299

’o’4 ↑= ·96059601
96059601

’o’5 ↑= ·9509900499
9509900499

’o’6 ↑= ·941480149401
941480149401

’o’7 ↑= ·932065347906 99

The result before obtained. In this way, by simple subtractions, Table II. may be constructed, filled up, and extended.

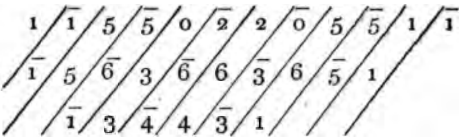
BY THE THIRD METHOD,

to eight places of decimals.

	1	0	0	0	0	0	0	0	0	· +	1
			7	0	0	0	0	0	0	· −	7
				2	1	0	0	0	0	· +	21
						3	5	0	0	· −	35
								3	5	· +	35
Positive	1	0	0	2	1	0	0	3	5	From	
Negative			7	0	0	3	5	0	0	Take	
’o’7 ↑=	·	9	3	2	0	6	5	3	5		

Ex. 6. Reduce ’11 ↑ to a common number.

The operative numbers for 11, being
1; 11; 55; 165; 330; 462; 462; 330; 165; 55; 11; 1, the work will stand thus,



’11 ↑= ·3 1 3 8 1 0 5 9 6 0 9

EXAMPLES FOR PRACTICE.

Ex. 8. Reduce ’o’o’7 ↑ to an ordinary number.

Ans. ·99302096503497900699 9.

Ex. 9. Reduce ’1 ↑; ’2 ↑; ’3 ↑; ’4 ↑; ’5 ↑; ’6 ↑; ’7 ↑; ’8 ↑; ’9 ↑ to ordinary numbers, and compare the results

with those given in Table II., and see if they agree to eight places of decimals.

Ans. ·9; ·81; ·729; ·6561; ·59049; ·531441; ·4782969; ·43046721; ·387420489.

PROBLEM II.

53. To find the common number answering to a dual number of two digits.

Rule.

Set down the ordinary number answering to either of the digits found by the last problem, and operate upon the result for the other given digit.

Examples.

Ex. 1. Reduce ↓ 3,5, to a common number true to eight places of decimals.

		E		D		C		B		A		
↓3,	=	1	3	3	1	0	0	0	0	0	·	1A.
				6	6	5	5	0	0	0	·	5B
					1	3	3	1	0	0	·	10C
↓²5,							1	3	3	1	·	10D
										7	·	5E
		1	·3	9	8	8	9	4	3	8		

5; 10; 10; 5; are multipliers easily operated with; beginning at B, C, D, E, respectively of the first horizontal row. 7 · is set down for 66 found in multiplying by the last 5, beginning at E.

OTHERWISE THUS:

↓0,5, = 1 0 5 0 0 0 0 0 5 0 1

1 1

	1	0	5	1	0	1	0	0	5	1
↓3,		3	1	5	3	0	3	0	2	3
			3	1	5	3	0	3	0	3
				1	0	5	1	0	1	1
↓3,5, =	1 ·	3	9	8	8	9	4	3	8	

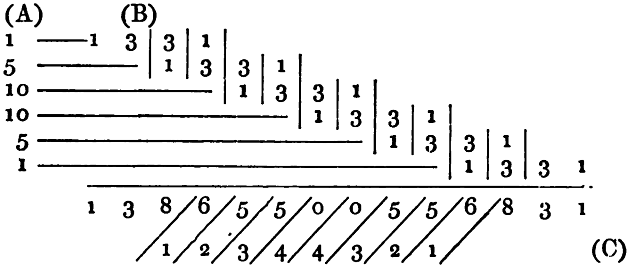
By common addition:

$\downarrow 1,$		1·1
		1·1
$\downarrow 2,$		1·2 1
		1·21
$\downarrow 3,$		1·3 3 1
		1·331
$\downarrow 3,1,$		1·3 4 431
		1·34431
$\downarrow 3,2,$		1·3 5 77531
		1·3577531
$\downarrow 3,3,$		1·3 7 1330631
		1·371330631
$\downarrow 3,4,$		1·3 8 504393731
		1·38504393731
$\downarrow 3,5,$	=	1·3 9 88943766831

∴ $\downarrow 3,5,$ 1·3 9 889438 true to eight places of decimals.

BY ANOTHER METHOD,

to be explained at the end of the work.



$\downarrow 3,5,$=1 · 3 9 8 8 9 4 3 7 6 6 8 3 1(D)

54. The multipliers (A) are combined in succession with the corresponding perpendicular rows of (B), thus producing the numbers of (C). Then (D) is found by addition. The first half of the numbers in (C) being found, the last half may be set down, since both halves are alike and symmetrical. The ultimate object of this chapter is to arrive at a simple, and at the same time a general, rule to convert a dual number of eight digits to a common number.

Ex. 2. Reduce ↓7,5, to a common number.

↓7,	1	7/1/.5/5/1/7 (2/3/3/2)							1	
	1	9	4	8	7	1	7	1	0	0
			9	7	4	3	5	8	5	5
↓0,5,				1	9	4	8	7	1	7
						1	9	4	8	7
									9	7
↓7,5, =	2 ·	0	4	8	1	2	1	2	5	6

The complete value of ↓7,5, is 2·04812125690726 71, and can be determined by the different methods just explained and illustrated.

Ex. 3. Reduce ↓0,0,0,0,6,8,0,0, ; ↓0,0,0,0,0,6,8,0, ; and ↓0,0,0,0,0,0,6,8, to common numbers.

	1 0000 | 0000 ·	1	
$\overset{5}{\downarrow}$ 6,	6000 ·	6	
	·	15	
$\overset{6}{\downarrow}$ 8,	1 00006 | 000...		1
	800...		8
↓0,0,0,0,6,8,0,0, =	1·00006 800		

55. The coefficients for $\overset{n}{\downarrow}$6, or 6, in any position being 1; 6; 15; 20; 15; 6; 1; and those for 8, in any position being 1; 8; 28; 56; 70; 56; 28; 8; 1. See the tabulated form (23), (49). However, in the present example, but 1; 6; of the one, and 1; 8; of the other set of coefficients, are operated with to arrive at the required degree of accuracy. It is also easily perceived that

↓0,0,0,0,0,6,8,0, = 1·00000680
and ↓0,0,0,0,0,0,6,8, = 1·00000068

Hence, with respect to the ascending branch, any dual number of four digits in the 5th, 6th, 7th, and 8th positions is converted to a common number by writing the dual number as a common number, and for the sign ↓ write 1· A single example will suffice to make this intelligible.

↓0,0,0,0,8,7,6,5, = 1·00008765

correct to eight places of decimals.

Ex. 4. Reduce '0'5 ↕ 3, to a common number.

↓3=1	·3	3	1	0	0	0	0	0	·+	1
		6	6	5	5	0	0	0	·−	5
'5 ₂↑			1	3	3	1	0	0	·+	10
					1	3	3	1	·−	10
								7	·+	5

'5 ²↕ 3, = 1 ·2 6 5 7 6 7 7 6

Second Method,

by common Addition and Subtraction.

```
 11
  11
 121
  121
↓3, = 1331
  1331
 131769
  131769
 13045131
  13045131
 1291467969
  1291467969
 127855328931
  127855328931
 12657677564169
```

'0'5 ↕ 3, = 1·26576776 true to eight places of decimals.

56. ## Third Method,

to be explained at the end of the work.

+ 1	1	3	3	1										
− 5			1	3	3	1								
+ 10					1	3	3	1						
− 10							1	3	3	1				
+ 5									1	3	3	1		
− 1											1	3	3	1
	1	3	$\bar{2}$	$\bar{4}$	$\bar{5}$	5	$\dot{0}$	$\bar{0}$	$\bar{5}$	5	4	2	$\bar{3}$	$\bar{1}$
			$\bar{1}$		2	2	$\bar{2}$	$\bar{2}$		1				(D)

'0'5 ↕ 3, = 1 · 2 6 5 7 6 7 7 5 6 4 1 6 9

The left half of (D) is symmetrical with the right half with contrary signs.

Ex. 5. Reduce '3'7 ↑ to a natural or common number.

The products by 7; 21; 35; &c. are very readily obtained, 21 being 3 times 7, and 35 five times 7.

First Method.

$'3\uparrow = 1\ \bar{3}\ 3\ 1$

	·7	2	9	0	0	0	0	0	+ 1
		5	1	0	3	0	0	0	− 7
'0'7 ↑			1	5	3	0	9	0	+ 21
					2	5	5	2	− 35
							2	6	+ 35
'3'7 ↑ =	·6	7	9	4	7	5	6	4	

Second Method,
by common Subtraction.

'3 ↑ = ·729
729
———
72171
72171
———
7144929
7144929
———
707347971
707347971
———
70027449129
70027449129
———
6932717463771
6932717463771
———
686339028913329
686339028913329
———
'3'7 ↑ = ·67947563862419571

The complete value of the given dual number, hence the first result is correct to eight places of decimals.

THIRD METHOD.

+1	1	0	$\overline{7}$	0	21	0	$\overline{35}$	0	35	0	$\overline{21}$	0	7	0	$\overline{1}$			
−3		1	0	$\overline{7}$	0	21	0	$\overline{35}$	0	35	0	$\overline{21}$	0	7	0	$\overline{1}$		
+3			1	0	$\overline{7}$	0	21	0	$\overline{35}$	0	35	0	$\overline{21}$	0	7	0	$\overline{1}$	
−1				1	0	$\overline{7}$	0	21	0	$\overline{35}$	0	35	0	$\overline{21}$	0	7	0	$\overline{1}$
	1	$\overline{3}$	$\overline{4}$	0	0	$\overline{6}$	8	4	$\overline{0}$	$\overline{0}$	4	8	$\overline{6}$	0	0	$\overline{4}$	$\overline{3}$	1
			2		5	2	8	7	7	8	2	$\overline{5}$		2				

'3'7↑ = · 6 7 9 4 7 5 6 3 8 6 2 4 1 9 5 7 1

Ex. 6. Reduce 'o'o'o'o'8'3'o'o↑; 'o'o'o'o'o'8'3'o↑ and 'o'o'o'o'o'o'8'3↑ to a common number.

SECOND CASE.

$$\begin{array}{rcl|l}
 & & 100000 & 000\ldots +1 \\
 & & & 800\ldots -8 \\
\hline
'8\underset{6}{\uparrow} & = & \cdot 9999920 & 0\ldots\ldots +1 \\
 & & 3 & 0\ldots\ldots -3 \\
\hline
'8\underset{6}{\uparrow}'3\underset{7}{\uparrow} & = & \cdot 9999917\ 0 &
\end{array}$$

FIRST CASE.

$$\begin{array}{rcl|l}
'8\underset{5}{\uparrow} & & 10000 & 0000.\ \ +1 \\
 & & & 8000.\ \ -8 \\
\hline
'3\underset{6}{\uparrow} & & \cdot 999920 & 00\ldots\ldots +1 \\
 & & 3 & 00\ldots\ldots -3 \\
\hline
'8\underset{5}{\uparrow}'3\underset{6}{\uparrow} & = & \cdot 999917\ 00 &
\end{array}$$

THIRD CASE.

$$\begin{array}{rl|l}
'8\underset{7}{\uparrow} & 1000000 & 00\ \ +1 \\
 & & 80\ \ -8 \\
\hline
'3\underset{8}{\uparrow} & \cdot 99999920 & +1 \\
 & 3 & -3 \\
\hline
 & \cdot 99999917 &
\end{array}$$

57. Hence, with respect to the descending branch, any dual number of four digits in the 5th, 6th, 7th, and 8th positions is converted to a common by writing the arithmetical complement of the given dual number as of a common number. A single example will make this matter intelligible.

EXAMPLE.

'o'o'o'o'3'5'8'9↑ = the ordinary number ·99996411 true to eight *places of decimals.*

It is also evident that a dual number expressed by eig[ht] dual digits, four ascending and four descending, occupyi[ng] the 5th, 6th, 7th, and 8th positions in each branch, is r[e]duced to a common number by merely taking the du[al] digits of one branch from those of the other as if commo[n] numbers.

'0'0'0'0'6'7'8'9 ↕ 0,0,0,0,8 9,2,3,=1·00002134

For, 8923
6789
2134

·99997866='0'0'0'0'8'9'2'3 ↕ 0,0,0,0,6,7,8,9,

For, from 8923
take 6789
·00002134 take
1 ·00000000 from
·99997866

PROOF; FIRST REDUCTION.

↓ 0,0,0,0,8,9,2,3,=1·0000|8923 from
· · · † |6000 −6
700 −7
80 −8
9 −9
6789 take

'6'7'8'9 ↕ 8,9,2,3,=1·00002134 (with 5 above and below the arrow)

SECOND REDUCTION.

↑ 0,0,0,0,6 7,8,9,=1·0000 6789 from
· · · †|8000 −8
900 −9
20 −2
3 −3
8923 take

'8'9'2'3 ↕ 6,7,8,9,=·99997866

PROBLEM.

Ascending Branch.

58. To reduce to a common number the 4th, 5th, 6th, 7th, and 8th digits of a given dual number.

RULE.

Add to the dual number taken as a common number the third coefficient or operative number, counting the unit, belonging to the fourth digit, and the whole number produced by multiplying the last four digits, considered a decimal, by the 4th digit.

EXAMPLES.

Ex. 1. Reduce $\downarrow$ 0,0,0,6,4,3,2,7, to a common number.
The first three coefficients for 6 are

1 ; 6 ; 15.

$$\begin{array}{r} \text{And } \cdot 4327 \\ 6 \\ \hline 3\cdots\cdot \end{array}$$

The whole number 3 is found without performing the multiplication designated

$$\begin{array}{r} 1\cdot 00064327 \\ 15 \\ 3 \\ \hline 1\cdot 00064345 \end{array} = \overset{4}{\downarrow} 6,4,3,2,7,$$

Ex. 2. Reduce $\downarrow$ 0,0,0,8,7,6,5,4, to a natural number.
The coefficients for 8 are 1 ; 8 ; 28 ;

$$\begin{array}{r} \text{and } \cdot 7654 \\ 8 \\ \hline 6\cdots\cdot \end{array}$$

$$\begin{array}{r} \text{Then } 1\cdot 00087654 \\ 28 \\ 6 \\ \hline 1\cdot 00087688 \end{array} = \overset{4}{\downarrow} 8,7,6,5,4,$$

Ex. 3. Reduce $\downarrow$ 0,0,0,5,6,7,8,9, to an ordinary number.

$$\begin{array}{r} \downarrow 0,0,0,5,6,7,8,9 \\ 10 \\ 3 \\ \hline 1\,0\,0\,0\,5\,6\,8\,0\,2 \end{array} = \overset{4}{\downarrow} 5,6,7,8,9$$

Problem.

Descending Branch.

59. To reduce to a common number the 4th, 5th, 6th, 7th, and 8th digits of a given dual number.

Rule.

Subtract from the dual number, taken as a common number, the third coefficient belonging to the fourth dual digit, and the whole number produced by multiplying the last four digits, considered a decimal, by the 4th digit. Then subtract the whole from 1·00000000, the remainder is the natural number required.

Examples.

Ex. 1. Reduce $'0'0'0'7'6'3'4'3\uparrow$ to a common number.

The first three coefficients of 7 are

1 ; 7 ; 21 ;

And ·6343

7

4····

·00076343

25

·00076318

$\cdot 99923682 = '7'6'3'4'3\uparrow_4$

Ex. 2. Reduce $'0'0'0'4'9'3'6'7\uparrow$ to a natural number.

Coefficients for 4 are 1 ; 4 ; 6 ;

·9367

4

4····

·00049367

4

6

·00049357

$\cdot 99950643 = '4'9'3'6'7\uparrow_4$

The chief object of this chapter is the concise and practical solution of the next problem; many preparatory processes have been introduced and exemplified, so that the methods employed in the solution may be easily understood and remembered.

PROBLEM.

60. To reduce a dual number of eight digits to a common number.

RULE I.

Reduce the last five digits to a common number; then, beginning with this number, operate in succession for each of the three remaining digits, in any order whatever, observing to divide the successive results counting from left to right into periods of *three figures*, when operating for the third digit; into periods of *two figures*, when operating for the second digit; and into periods of *single figures*, when the reduction is for the first digit.

EXAMPLES.

Ex. 1. Reduce ↓ 3,2,5,6,7,8,4,9, to a common number.

		↓0,	0,	0,	6,	7,	8,	4,	9,	The last five digits
								1	5	reduced to a com-
									5	mon number.
	1	0	0	0	6	7	8	6	9	
		3	0	0	2	0	3	6	1	
↓ 3,			3	0	0	2	0	3	6	
				1	0	0	0	6	8	
	1	·3	3	1	9	0	3	3	4	
$\overset{3}{\downarrow}$5,				6	6	5	9	5	2	
						1	3	3	2	
									1	
	1	3	3	8	5	7	6	1	9	·
$\overset{2}{\downarrow}$2,			2	6	7	7	1	5	2	·
					1	3	3	8	6	·
∴	1	·3	6	5	4	8	1	5	7	= ↓ 3,2,5,6,7,8,4.9,

Ex. 2. Reduce 10 $\overset{3}{\downarrow}$ 1,7,1,3 9 7,6,8, to a natural number.

									3	
	↓0,	0,	0,	3,	9,	7,	6,	8,		
									3	
	1	·0	0	0	3	9	7	7	4	·
			7	0	0	2	7	8	4	·
$\overset{2}{\downarrow}$7,				2	1	0	0	8	4	·
						3	5	0	1	·
								3	5	·
	1	0	7	·2	5	.6	1	7	8	

```
1  0  7  2  5  6  1  7  8      ↓1,
   1  0  7  2  5  6  1  8
                                 3
1  1  7  9  8  1  7  9  6      ↓1,
         1  1  7  9  8  2
                                 3
1  1  8  0 ·9  9  7  7  8 = 10↓1,7,1,3,9,7,6,8,
```

Ex. 3. Reduce '1'2'5'7'6'3'4'3 ↑ 10^4 to a common number.

```
      '0  '0  '0  '7  '6  '3  '4  '3↑
                              2   1 }
                                  4 } subtract
      ·0   0   0   7   6   3   1   8
      ·9   9   9 | 2   3   6 | 8   2   · Complement to 1.
'5↑            4 | 9   9   6 | 1   8   ·—
  3                        9 | 9   9   ·+
                                   1   ·—
      ·9   9 | 4   2 | 5   0 | 6   3+
'2↑        1 | 9   8 | 8   5 | 0   1—
 ·2                  | 9   9 | 4   3+
      ·9   7   4   4   6   5   0   5+
           9   7   4   4   6   5   1—
       8   7   7   0 · 1   8   5   4 = '1'2'5'7'6'3'4'3 ↑ 10
```

Ex. 4. Reduce '8'1'3 ↑ 10^6 to a natural number.

```
'8↑     1   8̄   8   6̄   0   6̄   8   8̄   1
              2   5̄   7   5̄   2

'1↑        ·4   3   0   4   6   7   2   1
  2                 4   3   0   4   6   7
            4   2   6 | 1   6   2 | 5   4   . +
                    1 | 2   7   8 | 4   9   . —
                              1 | 2   8   . +
            4   2   4   8   8   5 · 3   3     = '8'1'3 ↑ 10⁶
```

PROBLEM.

61. To reduce a dual number of eight digits to a common number.

RULE II.

Reduce the last five and the first three dual digits to common numbers separately, then multiply these numbers

together by common contracted multiplication, the product will be the required common number.

EXAMPLES.

Ex. 5. Reduce ↓ 7,2,6,0,7,8,2,6, to a common number.

↓ 7,	1	7	1	5	5	1	7	1		
		2	3	3	2					
$\overset{3}{\downarrow}$ 6,	1 ·	9	4	8	7	1	7	1	0	
			1	1	6	9	2	3	0	
						2	9	2	3	
									4	
$\overset{2}{\downarrow}$ 2,	1 ·	9	6	0	4	3	8	6	7	·
			3	9	2	0	8	7	7	·
					1	9	6	0	4	·
	1 ·	9	9	9	8	4	3	4	8	
Contr. Mult. }	6	2	8	7	0	0	0	0 ·	1	= 1·00007826 Inverted
	1 ·	9	9	9	8	4	3	4	8	
					1	3	9	9	9	
						1	6	0	0	
								4	0	
								1	2	
∴ (8), 2. or	1 ·	9	9	9	9	9	9	9	9	= ↓ 7,2,6,0,7,8,2,6,

Ex. 6. Reduce '10'3'7'4'2'3'4'5 ↑ 10^3 to an ordinary number.

'0'0'0'4'2'3'4'5 ↑
6
1
·0 0 0 4 2 3 3 8

'10 ↑	1	$\bar{0}$	5	$\bar{0}$	0	$\bar{2}$	0	$\bar{0}$	5	$\bar{0}$	1	
	$\bar{1}$	4	$\bar{2}$	1	$\bar{5}$	1	$\bar{2}$	4	$\bar{1}$			
	$\bar{1}$	2	$\bar{2}$	2	$\bar{1}$							
$'\underset{2}{3}$ ↑	· 3	4	8	6	7	8	4	4	0	1+		
		1	0	4	6	0	3	5 −				
				1	0	4	6	0 +				
							3	5 −				
	· 3	3	8	3	2	2	3	4				

```
          3  3  8 | 3  2  2 | 3  4 · +
'7₃↑            2 | 3  6  8 | 2  6 · —
                          7 | 1  0 · +
                            |    1 · —
          ----------------------------
          3  3  5  9 | 5  3  1  7 From—   the mult. by
          .  .  .  . |  -----------
                   1 | 3  4  3  8  — 4    ·00042338
                          6  7  2  — 2
                          1  0  1  — 3
                             1  0  — 3
                                3  — 8
                    --------------
                    1  4  2  2  4 Take—
          ----------------------------
          3  3  5 ·8  1  0  9  3=10'3'7'4'2'3'4'5 ↑ 10³
```

62. To reduce a dual number of any given number of digits to a common number.

Rule III.

Reduce the dual digits *seriatim* to the corresponding common number, dividing the result at each successive step into periods agreeing (21) (22) (51) with the position of each digit as the reduction is being made for that digit.

Examples.

Ex. 7. Find the common number represented by
↓ 0,3,2,1,3,3,0,5,7,7,7,8,5,3,

```
            10|00|00|00|57|77|85|3
  ²           |30|00|00|01|73|33|6
↓ 3,             |30|00|00|01|73|3
                    |10|00|00|00|6
         ---------------------------
  ³          103|030|105|952| 928
↓ 2,            |206|060|211| 906
                    |103|030| 106
         ---------------------------
  ⁴          1032|3626|9194| 940
↓ 1,             |1032|3626| 919
         ---------------------------
  ⁵          10324|65928|21859
↓ 3,              |30973|97785
                        |30974
         ---------------------------
  ⁶          103249|690250|618
↓ 3,               |309749|071
                          |310
         ---------------------------
         1·03249999999999 or 1·0325, (8)
```

63. Different methods of reduction are here introduced, each extremely simple, yet cases will occur when one of them will have the preference. A dual number multiplied by an ordinary number can be brought by similar means to an ordinary number. Proper examples fully worked out place the leading points of each method in a clear light, so that the practical bearing of each is easily observed.

Ex. 8. Find the value of 12·34 ↓ 2,3,4, in common numbers true to twelve places of figures.

1 2 3 4 0 0 0 0 0 0 0 0
↓ 2,   2 4 6 8 0 0 0 0 0 0 0
1 2 3 4 0 0 0 0 0 0

1 4 9 3 1 4 0 0 0 0 0 0
4 4 7 9 4 2 0 0 0 0
$\overset{2}{\downarrow}$ 3,   4 4 7 9 4 2 0 0
1 4 9 3 1 4

1 5 3 8 3 8 3 6 3 5 1 4
6 1 5 3 5 3 4 5 4
$\overset{3}{\downarrow}$ 4,   9 2 3 0 3 0
6 1 5

1 5·4 4 5 4 6 4 0 6 1 3

See *Ex.* 2. Page 19.

Ex. 9. Find the value of '2'3 ↑ $10^5$2 ↓ 5,5, in common numbers.

+1 | —1 5 10 10 5 1
—2 | —— 1 5 10 10 5 1
+1 | ——— 1 5 10 10 5 1

1 3 1 $\bar{5}$ $\bar{5}$ 1 3 1

+1 | —1 3 0 4 5 1 3 1
—3 | ———1 3 0 4 5 1 3 1
+3 | —————1 3 0 4 5 1 3 1
—1 | ———————1 3 0 4 5 1 3 1

1 3 3 $\bar{5}$ 8 $\bar{2}$ 3 $\bar{7}$ 6 $\bar{4}$ 4 2 $\bar{3}$ $\bar{1}$
$\bar{1}$

'2'3 ↕ 5,=   1 2 6 5 7 6 7 7 5 6 4 1 6 9
6 3 2 8 8 3 8 7 8 2
1 2 6 5 7 6 7 7 6
1 2 6 5 7 6 8
6 3 2 9
1 3

1 3 3 0 3 3·4 6 3 3 0 9

'2'3 ↑ 10^5 2 ↓ *5,5,* = *2 6 6 0 6 6·9 2 6 6 1 8* twice.

See example 5, page 18, where 133033·4608... is found by common addition and subtraction, (20).

EXAMPLES FOR PRACTICE.

Ex. 10. Reduce

$$'0'0'0'3'0'2'2'0'0'2'5'7'9'5 \updownarrow 0,3,0,0,1,0,5,5,$$

to a common number. *Ans.* 1·03

Ex. 11. Reduce

$$'0'0'1'0'0'1'0'1'0'0'6'2'3'2 \updownarrow 0,5,0,0,4,0,0,2,0,5,$$

to a common number. *Ans.* 1·05

Ex. 12. Find the value of $'4'3'2 \uparrow 47{\cdot}35$ in common numbers. See example 3, page 16. *Ans.* 30·0832669766

Ex. 13. Find the value of

$$'0'0'1'4'3'3'0'0'6'5'0'8'5'6 \updownarrow 0,6,0,0,0,4,5,$$

in common numbers. *Ans.* 1·06

Ex. 14. Find the value of

$$'0'0'0'6'0'0'6'0'0'1'5'7'3'7 \updownarrow 0,4,0,0,2,0,0,2,$$

in common numbers. *Ans.* 1·04

CHAPTER III.

64. TO REDUCE COMMON TO DUAL NUMBERS WITHOUT THE USE OF TABLES.

THE chief problem of this chapter is the *inverse* of that of the last; the solution of the *inverse* presents no difficulty, the *direct* operations being understood from the nature and flexibility of dual developments. *Auguste Comte*, in his "*Philosophy of Mathematics*," truly says, in summing up some of his general inquiries, "We have determined, at the beginning of this chapter, wherein properly consists the difficulty which we experience in putting mathematical questions into equations. It is essentially because of the insufficiency of the very small number of analytical elements which we possess, that the relation of the concrete to the abstract is usually so difficult to establish. Let us endeavour now to appreciate in a philosophical manner the

general process by which the human mind has succeeded, in so great a number of important cases, in overcoming this fundamental obstacle.

" *First, by the Creation of New Functions.*—In looking at this important question from the most general point of view, we are led at once to the conception of one means of facilitating the establishment of the equations of phenomena. Since the principal obstacle in this matter comes from the too small number of our analytical elements, the whole question would seem to be reduced to creating new ones. But this means, though natural, is really illusory; and though it might be useful, it is certainly insufficient. In fact, the creation of an elementary abstract function, which shall be veritably new, presents in itself the greatest difficulties.

" There is even something contradictory in such an idea; for a new analytical element would evidently not fulfil its essential and appropriate conditions, if we could not immediately determine its value. Now, on the other hand, how are we to determine the value of a new function which is truly simple, that is, which is not formed by a combination of those already known? That appears almost impossible. The introduction into analysis of another elementary abstract function, or rather another couple of functions, for each would be accompanied by its inverse, supposes then of necessity the simultaneous creation of a new arithmetical operation, which is certainly very difficult." The art and science of dual arithmetic supply all these requirements.

65. TO REDUCE COMMON TO DUAL NUMBERS.

RULE.

Take the common number corresponding to a dual digit of either branch, so that the leading figures of this number may approach the leading figures of the given number; then the dual digits, which have to be applied to bring the number selected to the given one, are the other digits of the required dual number.

In all dual developments, it should be remembered that a great many dual numbers can be found to represent the same common number. In reducing a common to a dual number, the work may be often abridged by multiplying or dividing either the number selected or the given number by 2, 4, or 8.

EXAMPLES.

1. Reduce 1234·56789 to a dual number.

It is not necessary to attend to the position of the decimal point until the work is completed.

```
↓2,  =        12|10|00|00|00
                |24|20|00|00
 2              |  |12|10|00
↓2,          ------------------
 4           1234|3 2 1 0|0 000
↓2,              |2 4 6 8|6 420
                 |       |1 234
 8          ---------------------
↓1,          1234 5 6 7 8|7 654  Take from given No.
             †           |-----
                         | 1346  Diff.
                         | 1235
             ---------------------
               10          |111
              ↓9,          |111
                           -----
```

Then $10^3 \downarrow$ 2,2,0,2,0,0,0,1,0,9,=1234·56789

66. When the dual number is restricted to nine positions, this number becomes,

$10^3 \downarrow$ 2,2,0,2,0,0,0,1,1,

the proper allowance being made for the digit rejected, (6), (35).

2. Reduce 987·654321 to a dual number.

```
'0'1↑        ·990|000|000|000+
 2↑             1|980|000|000−
 3                |990|000+
             ----------------------
             9880|2099|0000+
'3↑             2|9640|6297−
 4                  2|9641+
                     |   1−
             ----------------------
             98772|46133|43+
'7↑              6|91407|23−
 5                     20|74+
             ----------------------
'1↑          987655|474694
 6                 |987655
             ----------------------
             987654 487039  Take given N. from
             ....†  ------
       '1↑         1|66039  Diff.
        7           |98765
                    -------
        '6↑        6|7274
          8
```

$'6 \uparrow_{8}$	6|7274
	5|9259
$'8 \uparrow_{9}$	8|015
	7|901
$'1 \uparrow_{10}$	1|14
	|98
$'1 \uparrow_{11}$	1|6
	1|0
$'6 \uparrow_{12}$	6|
	6|

Then 987·654321 = '0'1'2'3'7'1'1'6'8'1'1'6 $\uparrow 10^3$

TO FIND EACH CONSECUTIVE DUAL DIGIT AFTER THE FIRST IS SELECTED.

Rule.

☞67. Take the difference between the given number and the result last found, then observe what multiple, the left hand figures of the step completed, is near this difference; that multiple gives a convenient dual digit to occupy the next position in order.

In the last example, having found the digits '0'1'2'3 ···· the result is 987724613343, therefore according to the rule

Result,	98772|4 ·····	
Given number,	98765|4 ·····	
98 ··)	7|0	Diff. (7 times, about.

68. Then '7 may be taken as the dual digit to occupy the fifth position, as we have no occasion to turn back and try another number, for then the method would be merely a method of approximation. Whether the results arrived at be greater or less than the given number, the process may be continued, since digits of either branch may be introduced at any stage of the reduction. To illustrate this *very important property*, let the digits '0'1'2'4 ··· be taken instead of '0'1'2'3 ··· then it will be seen, by the following reduction, that the result arrived at will be 987625840881, which is less than the given number; then the next digit must belong to the ascending branch;

Given number,	98765|43 ·····	
Result,	98762|58 ·····	
98 ···)	2|85	Diff. (3, times.
	2|94	

$'1\uparrow_{2}$ = ·990|000|000|000 +
$'2\uparrow_{3}$ 1|980|000|000 −
|990|000 +

9880|2099|0000 +
$'4\uparrow_{4}$ 3|9520|8396 −
5|9281 +
| 4 −

98762|5840|881 +
2|9628|775 + $\downarrow^{5}3,$
|296 +

987655 469952 From ; take given N.
····· 1|148952 Remainder
$'1\uparrow_{6}$ |987655
$'1\uparrow_{7}$ 1|61297 Contr. comm. div.,
|98765 987654 being the divisor
$'6\uparrow_{8}$ 6|2532
5|9259
$'3\uparrow_{9}$ 3|273
2|963
$'3\uparrow_{10}$ 3|10
2|96
$'1\uparrow_{11}$ 1|4
|9
$'5\uparrow_{12}$ 5|
5|

∴ ·987·654321 is also equal to

$'0'1'2'4'0'1'1'6'3'3'1'5 \updownarrow 0,0,0,0,3,$

Many dual numbers may be found to represent the same common number. Those of the present example, for nine places of figures, become

$'0'1'2'4'0'1'1'6 \uparrow 10^{8} \downarrow 0,0,0,0,3,$ and $'0'1'2'3'7'1'1'7 \uparrow 10^{8},$ not more than eight consecutive positions of either branch being occupied by dual digits.

69. In similar reductions half the digits of the required dual number being found by contracted division; when the number is great, mental labour may be saved if 1,2,3 ···· 9 times the divisor be taken, which can be done as follows, with little more mental exertion than that required to write the results; multiplication and division by 2, and simple subtraction being the chief operations required. In the

second example the divisor is 987654; without placing these multiples in regular order, they are given in the order in which they are most conveniently set down, and numbered (I), (II), (III), &c. as follows:—

	3—2962962	(V)	
Add	1— 987654	(I)	
	2—1975308	(II)	
	4—3950616	(III)	
	8—7901232	(IV)	

Ten times (I).

half	9876540	
(VI)	3938270	—5

	9876540	
	3950616	
(VII)	5925924	—6

	9876540	
	2962962	
(VIII)	6913578	—7 times 987654.

	9876540	
	987654	
(IX)	8888886	—9 times 987654.

70. To apply the operative numbers in the ordinary way before explained (23·), (49), presents no difficulty, yet their application may be much simplified in many cases, as the succeeding instances will show. Passing the operative numbers (1 1); (1 2 1); (1 3 3 1); the next set in order is (1 4 6 4 1), the number for the coefficient 4 being found, the result for 6 can be found by a simple subtraction.

Examples.

Ex. 1. Multiply 453176842 by ↓ 0,0,4,

Take	Bring up	1812*7 set down temporarily.				
From		453	1*76	842	1	
		1	8 12	707	4	Operative
			2	719	6	numbers.
$\overset{8}{\downarrow}$6,				2	4	
					(1)	Not required.

Explanation.

Take	1812*7	One period off; 4 times.
From	4531*7	Two periods off; 10 times given No.
Diff.	2719	Six times, commencing two periods from the right.

Ex. 2. Multiply 874983768 by ↓ 0,4,

Take	Bring up	34 99 93 5*1 temporarily set down.					
From		87	49	83	7*6	8·	—1
		3	49	99	3 5	1·	—4
				24	9 9	0·	—6 Diff.
							—4
							—1

Explained thus.

Take 3499935*1 (One period off); 4 times.
From 8749837*6 Two periods off, mult. by 10.
Diff. 5249902*5
or 524990·

71. Hence, the result for 4 being found, the other results for the coefficients (**6 4 1) may be readily obtained. To operate with (1 5 10 10 5 1) is a simple matter, since any number as 3764·273 multiplied by 10 becomes 37642·73. To multiply the same number by 5 take half 37642·73, thus

2)37642·73
18821·365 = 5 times 3764·273.

72. An easy way to find results for the coefficients (1 6 15 20 15 6 1).

Examples.

Ex. 1. Find the value of 3456·78855 ↓ 0,6, to nine places of figures.

0, after Double	6 9 1 3 5$^{c}_{\bullet}$7 7 1 0 0 0 = 20 times	
Add direct and diagonally	1 7 2 8 3 9 4 2 7 5	
	| 3 4 | 5 6 | 7 8 | 8 5 | 5 0 |	Given number with periods completed.
	2$^{e}_{\bullet}$0 7 4 0 7 3 1 3$^{a}_{\bullet}$0 0 = 6 times	
	5 1 8$^{d}_{\bullet}$5 1 8 2$^{b}_{\bullet}$8 2 5 0 = 15 times.	

Hence, when the given number is set down and the periods balanced, the other numbers may be set down immediately. From the right of each in succession reject two, four, six, eight, &c. figures, then the results may be *placed in proper* order.

This separation is at $\dot{a}$, $\dot{b}$, $\dot{c}$, &c.

COMMON METHOD.

```
 e  d  c  b  a
34|56|78|85|50      1
 2|07|40|73|13      6 times, beginning at a
  5|18|51|82       15   "      "      " b
     6 91 36       20   "      "      " c
        5 19       15   "      "      " d
         | 2        6   "      "      " e
--------------
36 69·45 07 0·
```

Ex. 2. Find the value of 345678855 ↓ 0,0,6, to nine places of figures.

```
0, after Double    6*9 1 3 5 7 7 1 0 0    —20
          Half     1 7 2 8 3 9 4 2 7 5
Given number        3 4 5|6 7 8|8 5 5
                   2 0 7 4 0 7 3*1 3 0    —6
                     5 1 8 5.1 8 2 8 2 5  —15
```

Casting off 3, 6, 9, &c. figures from the right.

ORDINARY METHOD.

```
         c    b    a
  3
↓ 6,    345|678|855     1
          2|074|073     6 times, beginning at a
              5|185    15   "      "      " b
                  7    20   "      "      " c
       ------------
       347 758 120·
```

Ex. 3. Find the value of ’6 ↑ 345678·855 to nine places of figures.

```
Zero after Double (N)   6 9 1 3 5 7 7*1 0 0  =  20
            Half (N)   1 7 2 8 3 9 4 2 7 5
  Given number (N)      3 4 5.6 7 8 8 5 5    =   1
                       2 0 7 4 0.7 3 1 3*0   =   6
                         5 1 8 5 1 8.2 8*2 5 =  15
```

Casting off 1, 2, 3, 4, 5, 6 figures from the right in succession.

ORDINARY METHOD.

	f	*e*	*d*	*c*	*b*	*a*					
3	4	5	6	7	8	8	5	5	+	1	
2	0	7	4	0	7	3	1	3	−	6,	beginning at *a*
	5	1	8	5	1	8	2	8	+	15	,, ,, *b*
		6	9	1	3	5	7	7	−	20	,, ,, *c*
			5	1	8	5	1	8	+	15	,, ,, *d*
				2	0	7	4	1	−	6	,, ,, *e*
						3	4	6	+	1	,, ,, *f*
1	8	3	7	0	7·	9	1	6			

73. To find the arithmetical complement of a given number;—begin at the left and subtract each figure from 9, except the last figure on the right, which take from 10; the result with minus 1 (written $\bar{1}$), placed on the left is a number called the arithmetical complement.

$$\begin{array}{ll} \text{No.} & 207407313 \\ \text{Arith. com.} & \bar{1}792592687 \end{array} \Big\} \text{ Sum} = 0.$$

When dual digits of the descending branch are introduced, subtractions may be avoided by employing the arithmetical complements of the numbers to be subtracted.

When the numbers for the respective coefficients are found by the contracted process, the vertical lines employed to divide the figures into periods may be omitted, besides, the arithmetical complements of the numbers to be subtracted can be set down without further preparation. From these advantages the contracted methods will be preferred in many cases. 73.

$$\begin{array}{rll}
345678\,855 & & 1 \\
\bar{1}792592\,687 & \text{Ar. com. for} & 6 \\
51851\,828 & & 15 \\
\bar{1}3086\,423 & \text{Ar. com. for} & 20 \\
518\,518 & & 15 \\
\bar{1}79\,259 & \text{Ar. com. for} & 6 \\
346 & & 1 \\
\hline
183707{\cdot}916 & &
\end{array}$$

Ex. 4. Required the numbers that should be added to 199876436758· to produce the same result as to multiply it by $(1{\cdot}01)^6$, true to twelve places of figures; or which is the *same thing*, find the value of 199876438758 $\downarrow$ 0,6,

CONTRACTED PRELIMINARY OPERATIONS.

Twice, with 0 after	3997528.775160	20
Left oblique. Half	999382193790	
Given No.	.199876438758	1
Add direct	119.9258632 5.48	6
Add obliquely	29981.4658.1370	15

ADDITION.

	199876438758	1
	11992586325*	6
	299814658*	15
↓²6,	3997529*	20
	29981*	15
	120*	6
	0	1

Required product ·212172867371

74. A simple method to find results for the coefficients

(1 7 21 35 35 21 7 1).

This method, like those before given, will be immediately understood from its application to one or two examples.

Ex. 1. Find the value of '7 ↑ 1·45678979 to nine places of figures.

PRELIMINARY OPERATIONS,

requiring but multiplication by 3 and division by 2.

Given number	14.5678979	1
3 times placed a fig. to right.	437036927	
Diff.	1019.75286.3	7
3 times (7)	30592.585.89	21
Half (7)	509876.4.315	35

WORK.

	1·45678979*	1
	$\bar{1}$8980247 14* Ar. co.	7
	$\underline{3}$0592586*	21
	$\bar{1}$4901236* Ar. co.	35
'7↑	$\underline{5}$09876*	35
	$\bar{1}$69407* Ar. co.	21
	1020*	7
	$\bar{1}$85* Ar. co.	1
	·69677803	

Ex. 2. Find the value of 875231866 ↓ 0,0,7, true to nine places of figures.

PRELIMINARY OPERATIONS.

Given number	8 7 5 2 3 1 8 6 6	1
3 times moved a figure to right	2 6 2 5 6 9 5 5 9 8	
Diff.	6 1 2 6 6 2 3.0 6 2	7
3 times diff.	1 8 3 7 9.8 6 9 1 8 6	21
Diff. with o, half	3 0.6 3 3 1 1 5 3 1 0	35

WORK.

c	*b*	*a*		
875	231	866		1
6	126	623	*	7
·	18	380	*	21
		31	*	35
		0		35
881	376	900·		

75. This result may be more readily found by the ordinary method; namely, mult. (1) by 7 beginning at *a*; 3 × (7) = 21; and 5 × (7) = 35.

In many cases the ordinary method is the best.

76. To render these contractions complete, examples to illustrate short methods when operating with the coefficients (1 8 28 56 70 56 28 8 1) and (1 9 36 84 126 126 84 36 9 1) are here added, although in practice seldom more than the use of (8 28) and (9 36) is required.

EXAMPLES.

Ex. 1. Find the value of 168594788 ↓ 0,8, true to nine places of figures.

PRELIMINARY ARRANGEMENT.

	n from (8) with o.	1 1 8.0 1 6 3 5 1 6 0	70
	Put *n* =	1 6 8 5 9 4 7 8 8	1
Adjacent Nos. operated upon	Twice *n*	3 3 7 1 8 9 5 7 6	
	Diff. obliquely taken	1 3 4 8 7 5 8 3.0 4	8
	Sum obliquely taken	4 7 2 0 6 5.4 0 6 4	28
	Double	.9 4 4 1.3 0 8 1 2 8	56

WORK.

$n =$	168594788	1
	13487583*	8
	472065*	28
	9441*	56
	118*	70
	1*	56
	183563996	

Ex. 2. Multiply 987654321234567890000 by $(\cdot 999)^8$; the product to be correct to 20 places of figures.

PREPARATORY ARRANGEMENTS.

n from (8); with 0 after		691 358 0248·64 197 523 0000	70
Sub. as indicated	$n=$	98 765 4321 23 456 789 000	1
Add, as indicated	$2n=$	197 530 8642 46 913 578 000	
		.790 123 4569 87 654 312.000	8
		2765.432 0994 56 790.092 000	28
		5530 864.1989 13.580 184 000	56

Then Add

987654321234567890000		1
$\bar{1}$209876543012345688*	Ar. co.	8
27654320994567 90*		28
$\bar{1}$44691358 01086*	Ar. co.	56
6913580249*		70
$\bar{1}$4469136*	Ar. co.	56
2765*		28
$\bar{1}$9*	Ar. co.	8
979680685746124444713		

The addition is the chief operation, the preparatory operations being little more than arrangement.

Ex. 3. Find the value of $8{\cdot}35768964 \downarrow 9$, true to nine places of figures.

PRELIMINARIES.

4 times		3008.768 27.04	36
		75.219 2067.6	9
Subtract	Giv. No.	835768 9640	
Add	Giv. No.	835768964	1
	Half	417884 4820	
Take from (9)		501461 3784	
		70204.592.976	84
	Half	35102296488	
		1053068.8.9464	126

77. The dual digit 9 in any position of either branch can be applied to 835768964, when the above preparation is made; proper attention being paid to the changing of ***.... The same remark applies to the preceding contracted operations.

WORK.

8·35768964	1
7 52192068*	9
3 008768827*	36
70204593*	84
10530688*	126
1053069*	126
70205*	84
3009*	36
75*	9
1*	1

19·70699499 true to the last figure.

EXAMPLES FOR PRACTICE.

Ex. 1. Reduce 1·03 to a dual number of fourteen dual digits. *Ans.* '3'0'2'2'0'0'2'5'7'9'5 $\overset{2}{\underset{4}{\updownarrow}}$ 30,0,1,0,5,5,

Ex. 2. What dual number of twenty digits will reduce to 1·1 without involving 1·1 or any power of it?

Ans. '3'0'0'0'5 $\overset{2}{\underset{7}{\updownarrow}}$ 9,5,7,6,0,0,3,5,8,0,8,0,7,4,4,4,2,7,8,

The consecutive results are

$$\overset{2}{\downarrow}9,=1093685272684360901 00$$

$$\overset{3}{\downarrow}5,=10991646468428317960 1$$

$$\overset{4}{\downarrow}7,=10999342929586722252 1$$

$$'3_{7}\uparrow\ =10999339629784173356 4$$

$$\overset{5}{\downarrow}6,=10999999606661189839 9$$

$$\overset{8}{\downarrow}3,=10999999993666118133 97$$

$$\overset{9}{\downarrow}5,=10999999999166118113 30$$

$$'5_{11}\uparrow\ =10999999999111118113 34$$

$$\overset{10}{\downarrow}8,=10999999999999111811 294$$

$$\overline{888188706}$$

$$1\cdot1)\underline{8\ 8\ 8\ 1\ 8\ 8\ 7\ 0\ 6}\quad\left\{\begin{array}{l}\text{common}\\ \text{division.}\end{array}\right.$$
$$8,0,7,4,4,4,2,7,8,$$

Ex. 3. Reduce 1·0325 to a dual number of fourteen digits. *Ans.* $\downarrow$ 0,3,2,1,3,3,0,5,7,7,7,8,5,3,

Ex. 4. Reduce 1·09 to a dual number.

Ans. '0'1'0'1'0'1'6'5'0'2'8'1'4'4 $\updownarrow$ 1,0,1,0,2,

Ex. 5. Reduce 1·08 to a dual number.

Ans. '0'0'0'3'0'4'0'0'8'4'4'8'9'8'0 $\updownarrow$ 0,8,0,4,

Ex. 6. Reduce 1·07 to a dual number.

Ans. '0'0'2'0'0'3'0'6'6'7'8 $\updownarrow$ 0,7,0,0,1,0,4,0,0,0,0,0,3,3,

CHAPTER IV.

78. TO CONVERT A DUAL NUMBER INTO A DUAL LOGARITHM, WITHOUT THE USE OF TABLES.

Rule I.

When the dual number to be reduced is composed of digits belonging to both branches, reduce the digits of the ascending branch to the eighth position by Rule II., and the digits of the descending branch to the same position by Rule III.; the difference of these results is the dual logarithm required, and belongs to the side that gives the greater number. (38), (45), (46), (47).

Ascending Branch.

Rule II.

To the dual number, taken as a common number, add 31018 times the first digit, and 33 times the second; then subtract 5 times the first three digits, a cipher being supposed after each, the remainder is the dual logarithm.

Examples.

Ex. 1. Find the dual logarithm of $\downarrow 3,1,4,1,2,1,1,3,$

$$
\begin{array}{rrl}
 & \downarrow 3,1,4,1,2,1,1,3, & \\
 & 93054 & = 3 \text{ times } 31018 \\
 & 33 & = \text{once } 33 \\
\hline
 & 31505200 & \\
\text{Subtract} & 1505200 & = 5 \text{ times } 3,01,04,0 \\
\hline
\text{Dual logarithm} = & 30000000, &
\end{array}
$$

The common number answering to

$$\downarrow 3,1,4,1,2,1,1,3 = 1{\cdot}34985881 \ (60);$$

$$\therefore \quad \downarrow, (1{\cdot}34985881) = 30000000,$$

Ex. 2. Reduce $\downarrow 7,2,6,0,7,8,2,6,$ to a dual logarithm.

$$
\begin{array}{rrl}
 & \downarrow 7,2,6,0,7,8,2,6, & \\
 & 217126 & = 7 \text{ times } 31018 \\
 & 66 & = 2 \text{ times } 33 \\
\hline
 & 72825018 & \\
 & 3510300 & = 5 \text{ times } 7,0,2,0,6,0, \\
\hline
\textit{Dual logarithm} = & 69314718, &
\end{array}
$$

79. Rule II. may be applied under another form, which may be expressed thus:—

Add together the dual number taken as a common number, the first digit times 31018, the second digit times 33, and the arithmetical complement of the first three digits, with (0) after each, multiplied by 5; the sum will be the dual logarithm.

$$
\begin{array}{rl}
\downarrow \underline{7},2,6,0,7,8,2,6, & \\
\bar{1}\,6\,4\,8\,9\,7\,0\,0 & = \text{Ar. co. } 702060 \times 5 \\
2\,1\,7\,1\,2\,6 & = 7 \times 31018 \\
6\,6 & = 2 \times 33 \\
\hline
6\,9\,3\,1\,4\,7\,1\,8, &
\end{array}
$$

Ex. 3. Reduce ↓ 8,5,7,7,7,0,4,1, to a dual logarithm.

First method.

$$
\begin{array}{rl}
\downarrow 8,5,7,7,7,0,4,1, & \\
2\,4\,8\,1\,4\,4 & = 8 \times 31018 \\
1\,6\,5 & = 5 \times 33 \\
\hline
8\,6\,0\,2\,5\,3\,5\,0 & \\
4\,0\,2\,5\,3\,5\,0 & = 5 \times 8,05,07,0 \\
\hline
\text{Dual log.} = 8\,2\,0\,0\,0\,0\,0\,0, &
\end{array}
$$

Second method.

$$
\begin{array}{rl}
\text{Dual as com. No.} = \underline{8}5777041 & \\
\bar{1}5974650 & = \text{Ar. co. of } 5 \times 8,05,07,0 \\
248144 & = 8 \times 31018 \\
165 & = 5 \times 33 \\
\hline
\text{D. L.} = 82000000, &
\end{array}
$$

Descending Branch.

Rule III.

80. Add together the dual number taken as a common number, 5 times the first three dual digits, supposing a cipher after each, 36052 times the first digit, and 34 times the second, the sum will be the dual logarithm.

EXAMPLES.

Ex. 1. Reduce '2'3'4'5'6'7'8'9 ↑ 10^4 to a dual logarithm.

Dual number '2'3'4'5'6'7'8'9 ↑

$$
\begin{array}{rl}
1015200 & =5\times\text{'}20\text{'}30\text{'}40 \\
72104 & =2\times36052 \\
102 & =3\times34 \\
\hline
\text{Dual log.}= \text{'}24544195 & \\
230258509\times4 = 921034036, & = \downarrow, (10)^4 \\
\hline
896489841, & \text{Required logarithm.}
\end{array}
$$

Ex. 2. Reduce '6'6'0'6'8'2'0'2 ↑ 10^3 to a logarithm.

$$
\begin{array}{rl}
\text{'}6\text{'}6\text{'}0\text{'}6\text{'}8\text{'}2\text{'}0\text{'}2\uparrow & \\
3030000 & =5\times606000 \\
216312 & =6\times36052 \\
204 & =6\times34 \\
\hline
\text{'}69314718 & \\
3\times230258509, = 690775527, & =\downarrow, (10)^3 \\
\hline
621460809, &
\end{array}
$$

'6'6'0'6'8'2'0'2 ↑ = ·5 ∴ ↓, (500) = 621460809,

Ex. 3. Reduce the dual number

'0'0'4'2'0'1'1'2 ↕ 0,10,0,0,1,0,0,0,

=

1·1

to a dual logarithm.

$$
\begin{array}{lrcrl}
\text{add} & 200 & & 330 & \text{add} \\
 & \text{'}0\text{'}0\text{'}4\text{'}2\text{'}0\text{'}1\text{'}1\text{'}2 & \updownarrow & 0,10,0,0,1,0,0,0, & \\
 & & & 50000 & \text{subtract} \\
\hline
 & \text{'}420312 & & 9951330, & \\
 & & & \text{'}420312 & \\
\hline
 & & & 9531018 &
\end{array}
$$

∴ ↓, (1·1) = 9531018, or 1·1 = ↓0,0,0,0,0,0,0,9531018,

Ex. 4. Reduce the dual number

$$'0'0'0'0'4'5'0'0 \updownarrow 0,0,10,0,0,0,3,3, = 1{\cdot}01$$

to a dual logarithm.

$$'0'0'0'0'4'5'0'0 \updownarrow 0,0,10,0,0,0,3,3,$$

$$\begin{array}{r} 500 \\ \hline 999533, \\ '4500 \\ \hline 995033, \end{array} \quad \text{subtract}$$

$\therefore \quad \downarrow, (1{\cdot}01) = 995033, \text{ or } 1{\cdot}01 = \overset{8}{\downarrow} 995033,$

Ex. 5. Reduce the dual number

$$'0'0'0'0'0'0'4'5 \updownarrow 0,0,0,10,0,0,0,0, = 1{\cdot}001$$

to a dual logarithm.

$$'0'0'0'0'0'0'4'5 \updownarrow 0,0,0,10,0,0,0,0,$$

$$\begin{array}{r} 5 \\ \hline 99995, \\ '45 \\ \hline 99950, \end{array}$$

$\therefore \quad \downarrow, (1{\cdot}001) = 99950, \text{ or } 1{\cdot}001 = \overset{8}{\downarrow} 99950,$

Ex. 6. Reduce $'0'0'0'0'0'0'0'0{\cdot}45 \updownarrow 0,0,0,0,10,0,0,0,$ to a dual logarithm. $= 1{\cdot}0001$

$$\cdot 45 \overset{5}{\underset{8}{\updownarrow}} 10,0,0,0,$$

$$\begin{array}{r} \cdot 45 \\ \hline 9999,\cdot 55 = 10000, \end{array}$$

$\therefore \quad \downarrow, (1{\cdot}0001) = 10000, \text{ or } 1{\cdot}0001 = \overset{8}{\downarrow} 10000,$ very nearly.

81. Each of the bases may be expressed in terms of those succeeding it. The following tabulated developments are extended to twenty consecutive dual digits, and will be found useful when accurate results are required to any number of places of figures less than twenty.

POSITIONS, DESCENDING. POSITIONS, ASCENDING.

Dual digits, Descending. Dual digits, Ascending.

	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
'0	'0	'0	'0	'0	'0	'0	'0	'0	'0	'0	'0	'0	'0	'0	'0	'0	'0	'0	'0		0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	10,	0,	0,	0,	0,	0,	0,	0,	0,	0,
'0																		'4	'5		0,									10,										0,
'0																'4	'5		'0		0,								10,											0,
'0											*f*			'4	'5				'0		0,							10,				An								0,
'0												'4	'5						'0		0,						10,												3,	3,
'0							F			'4	'5								'0		0,					10,			A								3,	3,		0,
0'								'4	'5							'2	'2	9	'5		0,				10,								3,	3,						0,
0'						'4	'5					'2	'2	'9	'3	'0	'0	'1	'8		0,			10,							3,	3,								0,
0'				'4	'5			'2	'2	'7	'5	'1	'5	'0	'7	'6	'2	'7	'2		0,		10,				3	3,												0,
0'	'0	'0	'4	2'	'1	'1	'2	'0	'3	'4	'1	'8	'5	'0	'3	'4	'9	'2	'7		0,	10,	0,	0,	1,	0,	0,	0,	3,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,
							E					e								c								r				z								

$$\cdot 9 \downarrow 1,1,0,1,0,0,0,1,0,0,0,0,0,0,0,1,0,0,0,0,0, = 1.$$
$$\cdot 99 \downarrow 0,1,0,1,0,0,0,1,0,0,0,0,0,0,0,1,0,0,0,0,0, = 1.$$
$$\cdot 999 \downarrow 0,0,1,0,0,1,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0, = 1.$$
$$\cdot 9999 \downarrow 0,0,0,1,0,0,0,1,0,0,0,0,0,0,0,0,1,0,0,0,0,0, = 1.$$

&c.

82. Since the logarithm of $1\cdot = 0$, then for eight dual digits, the dual logarithm of $\cdot 9$ added to $\downarrow 1,1,0,1,0,0,0,1,$ reduced to a dual logarithm, $= 0$, written (38)

$$\downarrow, (\cdot 9) + \downarrow, 1,1,0,1,0,0,0,1, = 0.$$

Therefore, when the dual logarithm of $\downarrow 1,1,0,1,0,0,0,1,$ is found, the dual logarithm of $\cdot 9$ becomes known. In a similar way the dual logarithm of $\cdot 99$; $\cdot 999$; &c. may be ascertained.

Examples.

Ex. 1. Required the dual logarithm of $\cdot 9 = {}'1 \uparrow =$

$$\begin{array}{rr}
\downarrow, 1, = & 9531018, \\
\overset{2}{\downarrow}, 1, = & 995033, \\
\overset{4}{\downarrow}, 1, = & 10000, \\
\overset{8}{\downarrow}, 1, = & 1, \\
\hline
 & 10536052,
\end{array}$$

$$\therefore \quad \downarrow, 1,1,0,1,0,0,0,1, = \overset{8}{\downarrow} 10536052,$$
$$\therefore \quad \downarrow, 1,1,0,1,0,0,0,1, = 10536052,$$
$$\therefore \quad \downarrow, (\cdot 9) = {}'10536052;$$
$$\text{or} \quad \cdot 9 = {}'1'0'0'0'0'0'0'0 \uparrow = {}'0'0'0'0'0'0'0'10536052 \uparrow$$
$$= {}'10536052 \underset{8}{\uparrow}$$

Ex. 2. Required the dual logarithm of $\cdot 99$

$$\cdot 99 \downarrow 0,1,0,1,0,0,0,1, = 1$$

$$\begin{array}{rr}
\overset{2}{\downarrow}, 1, = & 995033, \\
\overset{4}{\downarrow}, 1, = & 10000, \\
\overset{8}{\downarrow}, 1, = & 1, \\
\hline
 & 1005034,
\end{array}$$

$$\therefore \quad \downarrow, (\cdot 99) = {}'1005034.$$

Ex. 3. Reduce '0'0'1 $\uparrow$ = ·999 to a dual logarithm.

$$\cdot 999 \downarrow 0,0,1,0,0,1,0,0,=1.$$

$$\therefore\ \overset{3}{\downarrow},\ 1, = \quad 99950,$$

$$\overset{6}{\downarrow},\ 1, = \quad \underline{\quad 100,}$$

$$100050,$$

$$\therefore\ \downarrow,\ (\cdot 999) = {}'100050,$$

Ex. 4. Find the dual logarithm ·9999 the fourth base of the descending branch.

$$\cdot 9999 \downarrow 0,0,0,1,0,0,0,1,=1,$$

$$\overset{4}{\downarrow}\ 1, = \overset{8}{\downarrow}\ 10000,$$

$$\overset{8}{\downarrow}\ 1, = \overset{8}{\downarrow}\ \underline{\quad\quad 1,}$$

$$10001,$$

$$\therefore\ \downarrow,\ (\cdot 9999) = {}'10000 \text{ very nearly } (47).$$

Ex. 5. Find the dual number registered for 1·01 in the preceding table.

```
1 0 0 0/0 0 5/0 0 0/0 0 0/0 0 2/0 0 0/0 0 0
      1/    4/    2/    1/    5/    1/    2/          ↓^8 1(
       /     /   1/    2/    2/    2/    1/
--------------------------------------------
1 0 1 0 0|4 5 1 2 0|2 1 0 2 5|2 2 1 0 1|2+
         |4 0 4 0 1|8 0 4 8 0|8 4 1 0 0|9-
'4_5 ↑             |6 0 6 0 2|7 0 7 2 1|2+
                             |4 0 4 0 1|8-
                                       |1+
--------------------------------------------
1 0 1 0 0 0 4|7 1 9 0 1 1 4|6 6 8 3 1 9 8
             |3 0 3 0 0 1 4|1 5 7 0 3 4 4       ↓^7 3
                           |3 0 3 0 0 1 4
--------------------------------------------
1 0 1 0 0 0 5 0|2 2 0 1 2 9 1 2|8 3 5 5 6
               |3 0 3 0 0 1 5 0|6 6 0 3 9       ↓^8 3
                               |3 0 3 0 0
--------------------------------------------
1 0 1 0 0 0|5 0 5 2 3 1|3 0 6 3 7 9|8 9 5+
'5_6 ↑     |5 0 5 0 0 2|5 2 6 1 5 6|5 3 2-
                       |0 1 0 0 0 5|0 5 2+
                                  1|0 1 0-
--------------------------------------------
1 0 1 0 0 0 0 0 0|2 2 9 7 9 0 2 2 7|4 0 5+
```

```
1 0 1 0 0 0 0 0 0 ¦2 2 9 7 9 0 2 2 7 |4 0 5+
                  |2 0 2 0 0 0 0 0 0 |4 6 0−
                                     |1 0 1+
─────────────────────────────────────────────
1 0 1 0 0 0 0 0 0 0 |2 7 7 9 0 2 2 7 0 4 |6+
                    |2 0 2 0 0 0 0 0 0 0 |6−
                                         |1+
─────────────────────────────────────────────
1 0 1 0 0 0 0 0 0 0 0 0 |7 5 9 0 2 2 7 0 4 1
      '7  ↑              |7 0 7
        11               ─────
         '5  ↑             |5 2 0
           12              |5 0 5
            '1  ↑          ─────
              13             |1 5 2
                             |1 0 1
               '5  ↑         ─────
                 14            |5 1 2
                               |5 0 5
                  '7  ↑        ─────
                    16           |7 7 0
                                 |7 0 7
                     '6  ↑       ─────
                       17          |6 3 4
                                   |6 0 6
                        '2  ↑      ─────
                          18         |2 8 1
                                     |2 0 2
                           '7  ↑     ─────
                             19        |7 9 0
                                       |7 0 7
                              '8  ↑    ─────
                                20       |8 3 0
                                         |8 0 8
                                         ─────
                                           2 2
```

∴

1·01

⇓

'0'0'0'4'5'0'0'2'2'7'5'1'5'0'7'6'2'7'8 ↕ combined with the scending digits 0,0,10,0,0,0,3,3,0,0,0,0,0,0,0,0,0,0,0,0,

In the table (81), page 72, many of the ciphers are omitted.

Ex. 6. Find a dual number of twenty consecutive digits, as to verify the value of (1·1), given in the preceding table 1), and give the whole work unabridged. Each of the bases volved must be less than (1·1).

```
1 0 0/0 5/0 0/0 0/0 2/0 0/0 0/0 5 0 0 0 1
  1/  4/  2/  1/  5/  1/  2/  4   1            ↓² 10,
        1/  2/  2/  2/  1/
─────────────────────────────────────────────
1 1 0|4 6 2|2 1 2|5 4 1|1 2 0|4 5 1|0 0 1+
```

```
        1 1 0|4 6 2|2 1 2|5 4 1|1 2 0|4 5 1|0 0 1+
             |4 4 1|8 4 8|8 5 0|1 6 4|4 8 1|8 0 4−
'4 ↑               |6 6 2|7 7 3|2 7 5|2 4 6|7 2 3+
  3                      |4 4 1|8 4 8|8 5 0|1 6 4−
                               |1 1 0|4 6 2|2 1 3+
        ------------------------------------------
        1 1 0 0 2|1 0 2 6 0|2 2 4 9 2|8 2 7 9 6|9
                 |1 1 0 0 2|1 0 2 6 0|2 2 4 9 2|8
        ------------------------------------------
        1 1 0 0|2 2 1 2|6 2 3 2|7 5 3 0|5 2 8 9|7+
'2 ↑           |2 2 0 0|4 4 2 5|2 4 6 5|5 0 6 1|1−
  4                    |1 1 0 0|2 2 1 2|6 2 3 2|8+
        ------------------------------------------
'1 ↑    1 1 0 0 0 0|1 2 2 9 0 7|7 2 7 7 6 4|6 1 4+
  6                |1 1 0 0 0 0|1 2 2 9 0 7|7 2 8−
        ------------------------------------------
'1 ↑    1 1 0 0 0 0 0|1 2 9 0 7 6 0|4 8 5 6 8 8 6+
  7                  |1 1 0 0 0 0 0|1 2 9 0 7 6 0−
        ------------------------------------------
        1 1 0 0 0 0 0 0 0 1|9 0 7 6 0 3 5 6 6|1 2 6
                           |3 3 0 0 0 0 0 0 0 5|7 2 3
                                               |3 3 0
        ------------------------------------------
        1 1 0 0 0 0 0 0 0|2 2 3 7 6 0 3 5|7 2 1 7 9+
'2 ↑                     |2 2 0 0 0 0 0 0 0|4 4 7 5 2−
  8                                        |1 1 0 0 0+
        ------------------------------------------
        1 1 0 0 0 0 0 0 0 0 0|3 7 6 0 3 5 3 8 4 2|7+
'3                           |3 3 0 0 0 0 0 0 0 0 1|1−
  10                                               |3+
        ------------------------------------------
'4 ↑    1 1 0 0 0 0 0 0 0 0 0 0|4 6 0 3 5 3 8 4 1 9
  11                           |4 4
                          (1·1)  2 0 3 5 3 8 4 1 9 0.
                                 ---------------------
                                 '1 '8 '5 '0 '3 '4 '9 '2 '7
```

$$\therefore$$

$$1{\cdot}1$$

$$\parallel$$

'0'0'4'2'0'1'1'2'0'3'4'1'8'5'0'3'4'9'2'7 $\updownarrow$ combined with th ascending digits 0,10,0,0,1,0,0,0,3,0,0,0,0,0,0,0,0,0,0,0,

The other dual numbers tabulated in (81) page 72, are readi found. It has been assumed that

$$(1{\cdot}1)' = \downarrow 1, \quad = \overset{8}{\downarrow} 9531018,$$

$$(1{\cdot}01)' = \downarrow 0,1, \quad = \overset{8}{\downarrow} 995033,$$

$$(1{\cdot}001)' = \downarrow 0,0,1, \quad = \overset{8}{\downarrow} 99950,$$

$$(1{\cdot}0001)' = \downarrow 0,0,0,1, \quad = \overset{8}{\downarrow} 10000,$$

But these equalities may be established in an independent and a direct manner with great ease. Thus, from the tabulated arrangement (81), established by simple and direct operations, may be taken the eight digits D E F, C B A.

$$\downarrow 0,0,0,1, = \text{'0'0'0'0'0'0'0'0}_{\cdot 4\cdot 5} \updownarrow 0,0,0,0,10,0,0,0,0, = \overset{8}{\downarrow} 9999{,}_5$$

$$\downarrow 0,0,1, = \text{'0'0'0'0'0'0'4'5} \updownarrow 0,0,0,10,0,0,0,0,0,$$

$$\downarrow 0,1, = \text{'0'0'0'0'4'5'0'0} \updownarrow 0,0,10,0,0,0,0,3,3,$$

$$\downarrow 1, = \text{'0'0'4'2'0'1'1'2} \updownarrow 0,10,0,0,1,0,0,0,$$

$$\downarrow 0,0,0,1, = \overset{8}{\downarrow} 9999{\cdot}_5 = \overset{8}{\downarrow} 10000, \quad (35)\ (6)$$

$$\therefore \quad \downarrow 0,0,0,10, = \overset{8}{\downarrow} 99995,$$

for, if $(1{\cdot}0001)^1 = (1{\cdot}00000001)^{9999\cdot 5}$

then $(1{\cdot}0001)^{10} = (1{\cdot}00000001)^{9999.5}$;

(55) $\text{'0'0'0'0'0'0'4'5} \uparrow = \text{'45} \underset{8}{\uparrow}$, and consequently

$$\text{'0'0'0'0'0'0'4'5} \updownarrow 0,0,0,10,0,0,0,0,0, = \text{'45} \underset{8}{\overset{8}{\updownarrow}} 99995, = \overset{8}{\downarrow} 99950,$$

$$\therefore \quad \downarrow 0,0,1, = \overset{8}{\downarrow} 99950,$$

$$\therefore \quad \downarrow 0,0,10, = \overset{8}{\downarrow} 999500,$$

and $\therefore \quad \downarrow 0,0,10,0,0,0,0,3,3, = \overset{8}{\downarrow} 999533,$

also $\text{'0'0'0'0'4'5'0'0} \uparrow = \text{'4500} \underset{8}{\uparrow}$ (55),

$$\therefore \quad \text{'0'0'0'0'4'5'0'0} \updownarrow 0,0,10,0,0,0,0,3,3, = \text{'4500} \underset{8}{\overset{8}{\updownarrow}} 999533, =$$

$$\overset{8}{\downarrow} 995033,$$

$$\therefore \quad \downarrow 0,1, = \downarrow 995033,$$

$$\therefore \quad \downarrow 0,10, = \overset{8}{\downarrow} 9950330,$$

and $\therefore \quad \downarrow 0,10,0,0,1,0,0,0, = \overset{8}{\downarrow} 9951330,$

Again, (81), '0'0'1 $\uparrow$ reduced to the eighth position is equal to $\downarrow$ 0,0,1,0,0,1,0,0, reduced to the eighth position, but taken negatively.

But $\downarrow$ 0,0,1,= $\overset{8}{\downarrow}$ 99950,

$\therefore$ (82) $\downarrow$ 0,0,1,0,0,1,0,0,= $\overset{8}{\downarrow}$ 100050,

$\therefore$ '0'0'1 $\uparrow$ ='100050 $\underset{8}{\uparrow}$

$\therefore$ '0'0'4 $\uparrow$ ='400200 $\underset{8}{\uparrow}$

and $\therefore$ '0'0'4'2'0'1'1'2 $\uparrow$ ='420312 $\underset{8}{\uparrow}$;

hence '0'0'4'2'0'1'1'2 $\updownarrow$ 0,10,0,0,1,0,0,0,=

'420312 $\overset{8}{\underset{8}{\updownarrow}}$ 9951330,= $\overset{8}{\downarrow}$ 9531018,

$\therefore$ $\downarrow$ 1,= $\overset{8}{\downarrow}$ 9531018,

83. Hence, the equalities (81), and the accompanying tabulated developments, can be directly and independently determined as circumstances may require. Consequently the dual logarithm of (1·1) expressed $\downarrow$, (1·1) = 9531018,; $\downarrow$, (1·01)=995033,;

$\downarrow$, (1·001)=99950,; $\downarrow$, (1·0001)=10000,.

Dual logarithms to a greater extent may be calculated and applied by little additional preparations; a single instance will illustrate this matter. For example, let it be required to establish a system of dual logarithms to eleven places of figures, to which the corresponding dual numbers must be of twelve consecutive dual digits, (37). From the developments (81), (DEF) (ABC), may be taken

$\overset{6}{\downarrow}$ 1,='0'0'0'0'0'0'0'0'0'0'0'0$_{45}$ $\updownarrow$ 0,0,0,0,0,0,10,0,0,0,0,0,=

$\overset{12}{\downarrow}$ 999999,$_5$

$\overset{5}{\downarrow}$ 1,='0'0'0'0'0'0'0'0'0'0'4'5 $\updownarrow$ 0,0,0,0,0,10,0,0,0,0,0,0,

$\overset{4}{\downarrow}$ 1,='0'0'0'0'0'0'0'0'4'5'0'0 $\updownarrow$ 0,0,0,0,10,0,0,0,0,0,0,0,$_2$

$\overset{3}{\downarrow}$ 1,='0'0'0'0'0'0'4'5'0'0'0'0 $\updownarrow$ 0,0,0,10,0,0,0,0,0,3,3,0,

$\overset{2}{\downarrow}$ 1,='0'0'0'0'4'5'0'0'2'2'7'5 $\updownarrow$ 0,0,10,0,0,0,3,3,0,0,0,0,

$\downarrow$ 1,='0'0'4'2'0'1'1'2'0'3'4'2 $\updownarrow$ 0,10,0,0,1,0,0,0,3,0,0,0,

Reduction.

$$\overset{6}{\downarrow} 1, = 999999,_{5}\,. \qquad \text{(First).}$$

$$\overset{6}{\downarrow} 10, = 9999995,$$
$$'4'5\underset{11}{\uparrow} = \quad '45 \qquad \text{(Second).}$$
$$\therefore \quad \overset{5}{\downarrow} 1, = 9999950,$$

$$\overset{5}{\downarrow} 10, = 99999500,_{3}$$
$$'4'5\underset{9}{\uparrow} = \quad '4500 \qquad \text{(Third).}$$
$$\therefore \quad \overset{4}{\downarrow} 1, = 99995000,_{3}$$

$$\overset{4}{\downarrow} 10, = 999950003,$$
$$\overset{10}{\downarrow} 3,3,0, = \quad 330,$$
$$'4'5\underset{7}{\uparrow} = \quad '450000, \qquad \text{(Fourth).}$$
$$\therefore \quad \overset{3}{\downarrow} 1, = 999500333,$$

$$\overset{3}{\downarrow} 10, = 9995003330,$$
$$\overset{7}{\downarrow} 3,3,0,0,0,0, = \quad 330000,$$
$$9995333330,$$
$$'4'5'0'0'2'2'7'5\underset{5}{\uparrow} = \quad '45002477\cdot5 \qquad \text{(Fifth).}$$
$$\therefore \quad \overset{2}{\downarrow} 1, = 9950330853,$$

It may be shown, as follows, that

$$'4'5'0'0'2'2'7'5\underset{5}{\uparrow} = '45002477\cdot5$$

$$\downarrow 0,0,0,0,1,0,0,0,0,1,0,0, = \quad 10000050,$$
$$,4$$
$$\therefore \quad '4\underset{5}{\uparrow} = \quad '40000200$$
$$\downarrow 0,0,0,0,0,1,0,0,0,0,0,1, = \quad 1000000\cdot5$$
$$5$$
$$\therefore \quad '5\underset{6}{\uparrow} = \left.\begin{array}{r} '5000002\cdot5 \\ '40000200 \\ '2275 \end{array}\right\} \text{add}$$
$$\therefore \quad '4'5'0'0'2'2'7'5\underset{5}{\uparrow} = '45002477\cdot5$$

$$\overset{2}{\downarrow} 1, \quad = \underline{9950330853,}$$

$$\overset{2}{\downarrow} 10, \quad = 99503308530,$$

$$.\ \overset{5}{\downarrow} 1, \quad = \quad 9999950,$$

$$3000, \quad \text{(Sixth).}$$

$$\downarrow 0,10,0,0,1,0,0,0,3,0,0,0, \ = \ \overline{99513311480,}$$

$$'0\,'0\,'4'2'0'1'1'2'0'3'4'2 \uparrow = \ \underline{'4203131676}$$

$$\therefore \quad \downarrow 1, \ = \ 953101798 04,$$

That $'0'0'4'2'0'1'1'2'0'3'4'2 \uparrow = '4203131676 \underset{12}{\uparrow}$

may be determined thus,

$$\downarrow 0,0,1,0,0,1,0,0,0,0,0,1, = 1000500333,_5$$

$$\underline{\qquad\qquad 4}$$

$$\therefore \quad '4 \underset{3}{\uparrow} = '4002001334$$

$$\downarrow 0,0,0,1,0,0,0,1,0,0,0,0, = 100005000,$$

$$\underline{\qquad 2}$$

$$\therefore \quad '2 \underset{4}{\uparrow} = \ '200010000 \ = \ '200010000$$

$$'1'1'2'0'3'4'2 \underset{6}{\uparrow} = \quad \underline{'1120342}$$

$$\text{and} \ \therefore \quad '4'2'0'1'1'2'0'3'4'2 \uparrow = '4203131676$$

add

84. It will save the time of an operator to register in a tabulated form 1, 2, 3, 4, 5, 6, 7, 8, 9, times each of these ultimate values; for then the numbers to be added may be set down from these multiples.

TABULATED MULTIPLES,

of $\downarrow 1,$; $\overset{2}{\downarrow} 1,$; $\overset{3}{\downarrow} 1,$; $\overset{4}{\downarrow} 1,$; $\overset{5}{\downarrow} 1,$; $\overset{6}{\downarrow} 1,$ reduced to the twelfth position, taken from reductions made to the twentieth position.

$\downarrow 1,$

1		953101798 04,
2		1906203596 09,
3		2859305394 13,
4	$\overset{12}{\downarrow}$	3812407192 17,
5		4765508990 22,
6		5718610788 26,
7		6671712586 30,
8		7624814384 35,
9		8577916182 39.

$\overset{2}{\downarrow} 1,$

1		995033085 3,
2		1990066170 6,
3		2985099256 0,
4	$\overset{12}{\downarrow}$	3980132341 3,
5		4975165426 6,
6		5970198511 9,
7		6965231597 2,
8		7960264682 5,
9		8955297767 9.

$\overset{3}{\downarrow}1,$

1		999500333,
2		1999000666,
3		2998500999,
4	$\overset{12}{\downarrow}$	3998001332,
5		4997501665,
6		5997001999,
7		6996502332,
8		7996002665,
9		8995502998,

$\overset{4}{\downarrow}1,$

1		999950000,
2		1999900001,
3		2999850001,
4	$\overset{12}{\downarrow}$	3999800001,
5		4999750002,
6		5999700002,
7		6999650002,
8		7999600003,
9		8999550003,

$\overset{5}{\downarrow}1,$

1		9999950,
2		19999900,
3		29999850,
4	$\overset{12}{\downarrow}$	39999800,
5		49999750,
6		59999700,
7		69999650,
8		79999600,
9		89999550,

$\overset{6}{\downarrow}1,$

1		1000000,
2		1999999,
3		2999999,
4	$\overset{12}{\downarrow}$	3999998,
5		4999998,
6		5999997,
7		6999997,
8		7999996,
9		8999996,

$\overset{12}{\downarrow},(2)=693147180560$, and $\downarrow,(10)=2302585092994$,

Like multiples of $'1\uparrow\;=\;'105360515658\underset{12}{\uparrow}$

$'1\underset{2}{\uparrow}\;=\;'10050335854\underset{12}{\uparrow}$

$'1\underset{3}{\uparrow}\;=\;'1000500334\underset{12}{\uparrow}$

$'1\underset{4}{\uparrow}='100005000\underset{12}{\uparrow}$; $'1\underset{5}{\uparrow}='10000050\underset{12}{\uparrow}$; $'1\underset{6}{\uparrow}='1000001\underset{12}{\uparrow}$, when required, may be found and tabulated in a similar manner. A single example will suffice to illustrate this extension.

Example.

Reduce '0'0'4'2'1'0'5'0'4'6'7'8 $\updownarrow$ 7,6,0,0,0,8,0,3,0,0,0,0, to a dual logarithm in the twelfth position.

$$\left.\begin{array}{rl} '4\underset{3}{\uparrow} = & 4002001336 \\ '2\underset{4}{\uparrow} = & '200010000 \\ '1\underset{5}{\uparrow} = & '10000050 \end{array}\right\}\text{Tab. Mult.}$$

$$\begin{array}{r} '504678 \\ \hline '4212516064 \end{array}$$

$$\left.\begin{array}{rr} \downarrow 7, = & 667171258630, \\ \overset{2}{\downarrow} 6, = & 59701985119, \\ \overset{6}{\downarrow} 8, = & 7999996, \end{array}\right\}\text{Tab. Mult.}$$

$$\begin{array}{r} 30000, \\ \hline 726881273745, \\ '4212516064 \\ \hline 722668757681, \end{array}$$

$\therefore$ $'0'0'4'2'1'0'5'0'4'6'7'8 \updownarrow 7,6,0,0,0,8,0,3,0,0,0,0,$

$$= \overset{12}{\downarrow} 722668757681,$$

and may be written

$$'0'0'4'2'1'0'5'0'4'6'7'8 \overset{12}{\updownarrow}, 7,6,0,0,0,8,0,3,0,0,0,0,$$

$$= 722668757681$$

Tabulated Multiples.

of $'1\uparrow$; $'1\underset{2}{\uparrow}$; $'1\underset{3}{\uparrow}$ &c. reduced to twelfth position.

$'1\uparrow$

1		105360515658,
2		210721031316,
3		316081546974,
4		421442062632,
5		526802578290,
6		632163093948,
7		737523609606,
8		842884125264,
9		948244640922,

$'1\underset{2}{\uparrow}$

1		100503335854,
2		201006671708,
3		301510007562,
4		402013343416,
5		502516679270,
6		603020015124,
7		703523350978,
8		804026686832,
9		904530022686,

$'1\underset{3}{\uparrow}$

1		1000500334,
2		2001000668,
3		3001501002,
4		4002001336,
5		5002501670,
6		6003002004,
7		7003502338,
8		8004002672,
9		9004503006,

$'1\underset{4}{\uparrow}$

1		100005000,
2		200010000,
3		300015000,
4		400020000,
5		500025000,
6		600030000,
7		700035000,
8		800040000,
9		900045000,

$'1\uparrow_5$

1		1 0 0 0 0 0 5 0
2		2 0 0 0 0 1 0 0
3		3 0 0 0 0 1 5 0
4		4 0 0 0 0 2 0 0
5		5 0 0 0 0 2 5 0
6		6 0 0 0 0 3 0 0
7		7 0 0 0 0 3 5 0
8		8 0 0 0 0 4 0 0
9		9 0 0 0 0 4 5 0

$'1\uparrow_6$

1		1 0 0 0 0 0 1,
2		2 0 0 0 0 0 2,
3		3 0 0 0 0 0 3,
4		4 0 0 0 0 0 4,
5		5 0 0 0 0 0 5,
6		6 0 0 0 0 0 6,
7		7 0 0 0 0 0 7,
8		8 0 0 0 0 0 8,
9		9 0 0 0 0 0 9,

CHAPTER V.

85. TO REDUCE A DUAL LOGARITHM OF THE ASCENDING BRANCH TO A DUAL NUMBER, WITHOUT THE USE OF TABLES.

If the given logarithm be greater than $\downarrow$, (2) or $\downarrow$, (10), subtract multiples of 69314718, or 230258509, until the remainder is less than either of these constants. When the remainder thus found does not consist of eight places of figures, establish eight places by prefixing ciphers to the left, then apply for the ascending branch.

Rule I.

Add once; twice; three times, &c. 500000, according as the first figure, on the left of the sum, becomes respectively 1; 2; 3; &c.; subtract 31018 times the first figure of the sum, which first figure must not alter in the operation, but reappear in the remainder. Then add once; twice; three times, &c. 5000, according as the second figure to the left of the sum becomes respectively, 1; 2; 3; &c.; subtract 33 times the second figure, which must not change in the operation, but reappear in the remainder.

Again add once; twice; three times, &c. 50, according as the third figure of the sum becomes respectively 1; 2; 3; &c.; and the dual logarithm is reduced to a dual number of eight digits. This rule is the reverse of Rule II. (78).

Examples.

Ex. 1. Reduce the dual logarithm 69314718, to a dual number of eight digits.

$$
\begin{array}{rl}
69314718, & \\
3500000+ & =7\times 500000 \\
\hline
72814718 & \\
217126- & =7\times 31018 \\
\hline
72597592 & \\
10000+ & =2\times 5000 \\
\hline
72607592 & \\
66- & =2\times 33 \\
\hline
72607526 & \\
300+ & =6\times 50 \\
\hline
\end{array}
$$

$\therefore\ \downarrow$, 7,2,6,0,7,8,2,6, =69314718,

Ex. 2. Reduce 230258509, to a dual number.

$$
\begin{array}{rr}
 & 230258509, \\
 & 207944154, =3\times 69314718, \\
\hline
 & 22314355, \\
500000\times 2= & 1000000+ \\
\hline
 & 23314355 \\
31018\times 2= & 62036- \\
\hline
 & 23252319 \\
5000\times 3= & 15000+ \\
\hline
 & 23267319 \\
33\times 3= & 99- \\
\hline
 & 23267220 \\
50\times 6= & 100+ \\
\hline
\end{array}
$$

$\overset{3}{2}\downarrow$, 2,3,2,6,7,3,2,0,

$\therefore\ \downarrow^{3}$, 2,3,2,6,7,3,2,0,=230258509,= $\uparrow$, (10).

Ex. 3. Reduce the dual logarithm 60000000, to a dual number.

$$
\begin{array}{rr}
\text{Dual log.} & 60000000, \\
 & 3000000+ \\
\hline
 & 63000000 \\
 & 186108- \\
\hline
 & 62813892 \\
 & 10000+ \\
\hline
 & 62823892 \\
 & 466- \\
\hline
 & 62823826 \\
 & 400+ \\
\hline
\end{array}
$$

$\downarrow$ 6,2,8,2,4,2,2,6, dual number.

Ex. 4. What dual number corresponds to the dual logarithm 136969042 2, ?

$$
\begin{array}{rr}
 & 1\,3\,6\,9\,6\,9\,0\,4\,2\,2, \text{ given log.} \\
\downarrow, 10^8 = & 1\,1\,5\,1\,2\,9\,2\,5\,4\,5 \\
\hline
 & 2\,1\,8\,3\,9\,7\,8\,7\,7 \\
\downarrow, 2^3 = & 2\,0\,7\,9\,4\,4\,1\,5\,4 \\
\hline
 & 1\,0\,4\,5\,3\,7\,2\,3 \\
1 \times 500000 = & 5\,0\,0\,0\,0\,0 \\
\hline
 & 1\,0\,9\,5\,3\,7\,2\,3 \\
1 \times 31018 = & 3\,1\,0\,1\,8 \\
\hline
 & 1\,0\,9\,2\,2\,7\,0\,5 \\
9 \times 50 = & 4\,5\,0 \\
\hline
 & {}_2\!\downarrow^3\ 1,0,9,2,3,1,5,5, \text{ dual number.}
\end{array}
$$

Ex. 5. What dual number corresponds to the dual logarithm 345676, ?

$$
\begin{array}{rr}
\text{Given } & 0\,0\,3\,4\,5\,6\,7\,6, \\
3 \times 50 = & 1\,5\,0 \\
\hline
\end{array}
\left\{ \begin{array}{l} \text{Given number with two} \\ \text{ciphers to the left to} \\ \text{make up eight places.} \end{array} \right.
$$

Dual number $\downarrow$ 0,0,3,4,5,5,2,6, = $\downarrow^3$ 3,4,5,5,2,6,

EXAMPLES FOR PRACTICE.

Ex. 6. Reduce the dual logarithm 30000000, to a dual number. *Ans.* $\downarrow$ 3,1,4,1,2,1,1,3,

Ex. 7. Find the dual number corresponding to the dual logarithm 82000000, *Ans.* $\downarrow$ 8,5,7,7,7,0,4,1,

Ex. 8. Reduce 230258509, to a dual number, without involving the logarithm of 2· *Ans.* $\downarrow$ 24,1,5,1,9,2,9,4,

86. To reduce a dual logarithm of the descending branch to a dual number.

RULE II.

Subtract once; twice; three times, &c., 536052 (*a*), according as the first figure on the left becomes 1; 2; 3; &c.; which first figure must not alter, but reappear in, the remainder.

Then subtract once; twice; three times, &c., 5034 (*b*), according as the second figure to the left of the remainder becomes respectively 1; 2; 3; &c.

Again, subtract once; twice; three times, &c., 50 (c), according as the third figure of the remainder becomes 1; 2; 3; &c., respectively. Thus the dual logarithm is reduced to a dual number of eight descending dual digits.

This Rule is the reverse of Rule III. (80).

Examples.

Ex. 1. Reduce 896489841, to a dual number of the descending branch.

$$\begin{array}{lrl}
 & +896489841, & \text{dual log.} \\
\downarrow, 10^4 & =-921034036 & \\
\hline
 & '24544195 & \\
 & 1072104 & =2a. \\
\hline
 & 23472091 & \\
 & 15102 & =2b. \\
\hline
 & 23456989 & \\
 & 200 & =2c. \\
\hline
\end{array}$$

Dual number '2'3'4'5'6'7'8'9 $\uparrow$ (10^4)

Ex. 2. Reduce '69314718 to a dual number.

$$\begin{array}{rl}
'69314718 & \text{dual logarithm} \\
3216312 & =6a. \\
\hline
66098406 & \\
30204 & =6b. \\
\hline
\end{array}$$

Dual number '6'6'0'6'8'2'0'2 $\uparrow$

87. Any dual logarithm may be compounded of multiples of 69314718 (n), and 230258509 (m), and a logarithm numerically less than 34657359 half the dual logarithm of 2.

If 230258509 alone is operated with, any logarithm may be compounded of multiples of 230258509 and a logarithm less than 115129255 half the logarithm of 10.

$\frac{1}{2}n=$	34657359	$\frac{1}{2}m=$	115129255
$n=$	69314718	$m=$	230258509
$1\frac{1}{2}n=$	103972077	$1\frac{1}{2}m=$	345387764
$2n=$	138629436	$2m=$	460517019
$2\frac{1}{2}n=$	173286795	$2\frac{1}{2}m=$	575646274
$3n=$	207944154	$3m=$	690775528
&c.		&c.	

If the given logarithm be greater than $\frac{n}{2}$ by x, but less than n, then

$$n-\left(\frac{n}{2}+x\right)=\frac{n}{2}-x,$$

a logarithm less than $\frac{n}{2}$.

If the given logarithm be greater than n, but less than $1\frac{1}{2}n$ by y, then

$$\left(\frac{3n}{2}-y\right)-n=\frac{n}{2}-y,$$

a logarithm less than $\frac{n}{2}$.

Again, if the logarithm be greater than $1\frac{1}{2}n$ by z, but less than $2n$, then

$$2n-\left(\frac{3n}{2}+z\right)=\frac{n}{2}-z,$$

which is also less than $\frac{n}{2}$, and so on. A similar process of reasoning may be applied to $\frac{1}{2}m$; m; $1\frac{1}{2}m$; $2m$; &c.

EXAMPLES.

Ex. 1. What multiples of the logarithms of 10· and 2· will bring the dual logarithm 178765437, less than 34657359,

$$\begin{array}{rl}
m=230258509 & \\
178765437, & \text{put}=L \\
\hline
51493072 & m-L \\
n=69314718 & \\
\hline
17821646, & \text{put}=R
\end{array}$$

$$\therefore\quad n-(m-L)=R, \text{ and } L=m-n+R$$
$$\text{or } \downarrow, (10)-\downarrow, (2) + 17821646.$$

$$\text{But (85), } \overset{8}{\downarrow} 17821646,=\downarrow 1,8,3,3,0,5,3,6,$$
$$=$$
$$1\cdot19508425$$
$$(10\times 1\cdot19508425\div 2)=5\cdot97542125$$
$$\therefore\quad \downarrow, (5\cdot97542125)= 178765437,$$

Ex. 2. Reduce '178765437 to a logarithm less than 34657359

$$
\begin{array}{rll}
m = & 230258509 & \\
 & 178765437 & \text{put} = L \\
\hline
 & 51493072 & = m - L \\
n = & 69314718 & \\
\hline
 & 17821646 & \text{put} = R
\end{array}
$$

$$n-(m-L)=R$$

$$\therefore\ n-R=m-L$$

$$\therefore\ n-m-R=-L$$

$$\therefore\ \downarrow,(2)+(10)\,'\!\uparrow+'17821646='178765437$$

Ex. 3. Reduce 98765432, and also '98765432 to a logarithm numerically less than 34657359

$$
\begin{array}{r}
98765432 \\
69314718 \\
\hline
9450714
\end{array}
$$

$$\therefore\ \downarrow,(2)+9450714,=98765432,\text{ and}$$

$$(2)\,'\!\uparrow+'9450714='98765432$$

$$98765432,$$

$$=$$

$$'0'0'1\ {}^{'}\!\!\updownarrow\!{}^{1}\ 1,0,0,1,9,7,4,6,$$

$$=$$

$$2{\cdot}202634$$

88. Because $\downarrow 3,6,0,9\ 4,1,0,7, = \overset{8}{\downarrow} 34657359,$

and $'3'3'0'3'4'1'0'1\uparrow = '34657359_{8}\uparrow$

hence, any dual logarithm may be reduced to a dual number whose first digit does not exceed 3, or '3; and by operating with the logarithms of $\downarrow 1,$; $\overset{2}{\downarrow} 1,$; $\overset{3}{\downarrow} 1,$; &c., and of $'1\uparrow$; $'1_{2}\uparrow$; $'1_{3}\uparrow$; &c., in a manner similar to that explained (87), with respect to '69314718 and 69314718, the succeeding digits may be found so as not to exceed 5, or '5.

Dual logarithms of $\downarrow 1,$ and $'1\uparrow$; $\overset{2}{\downarrow} 1,$ and $'1_{2}\uparrow$; $\overset{3}{\downarrow} 1,$ and $'1_{3}\uparrow$

$'1\,'\!\uparrow\ ='10536052$	$'1\underset{2}{\uparrow}\ ='1005034$	$1\underset{3}{\uparrow}\ ='100050$
$\downarrow,1,=\ 9531018,$	$\overset{2}{\downarrow}1,=\ 995033,$	$\overset{3}{\downarrow}1,=\ 99950,$

A multiple of 10536052 may be involved so that the remainder will not exceed half of 10536052=5268026 which contains 1005034 five times, but not six times; the same may be said of half 1005034 = 502517, &c. and of half 9531018=4765509; &c. &c.

Examples.

Ex. 1. Reduce 34657359, to a dual number, composed of digits, each digit not to be greater than 5, or '5

$$\begin{array}{r}
3\,4\,6\,5\,7\,3\,5\,9, \\
3\,8\,1\,2\,4\,0\,7\,2, = \downarrow 4, \\
\hline
'3\,4\,6\,6\,7\,1\,3 \\
'3\,0\,1\,5\,1\,0\,2 \\
\hline
'4\,5\,1\,6\,1\,1 \\
'4\,0\,0\,2\,0\,0 \\
\hline
5,1,4,1,1,
\end{array}$$

$$\therefore\ \ '0'3'4'5'1'4'1'1 \updownarrow, 4,=34657359,$$

The dual number just found is not as readily reduced to a common number as $'0'0'0'0'5'8'4'3 \updownarrow, 3,6,1,=34657359,$ and yet one of the digits $\overset{2}{\downarrow}6$, exceeds 5; but the first three digits $\downarrow 3,6,1$, in the latter case are more easily operated with than $'0'3'4 \updownarrow 4$, the first three in the former case, the reduction for 1, or '1 being an extremely simple operation; and whether the last four digits are greater or less than 5 is of no moment.

Reduction unabridged.

```
1 3|3 1|0 0|0 0|0
   |7 9|8 6|0 0|0      ↓3, 6,
      1|9 9|6 5|0
          2|6 6|2
              2|0
-------------------
1 4 1 2 8 8 3 3 2       3
      1 4 1 2 8 8      ↓1,
-------------------
1 4 1 4 2|9 6 2 0
```

```
          1 4 1 4 2|9 6 2 0
'5↑                |7 0 7 1—
 5   '8↑             1 1 3 1—
      6   '4↑            5 7—
           7   '3↑         4—
                8     8 2 6 3
       ∴   1·4 1 4 2 1 3 5 7
                  ||
'0'0'0'0'5'8'4'3 ↕ 3,6,1,=34657359,
```

Ex. 2. Reduce '34657359 to a dual number, each digit of which not to exceed 5, or '5

```
     '3 4 6 5 7 3 5 9
'3↑  '3 1 6 0 8 1 5 6
       '3 0 4 9 2 0 3   ∴  '34657359↑ ='3'3'0'3'4'1'0'1↑
       '3 0 1 5 1 0 2               8
           '3'4'1'0'1
```

REDUCTION.

$$
\begin{array}{l r}
\bar{1} & 1\bar{3}3\bar{1} \\
3 & 1\bar{3}3\bar{1} \\
\bar{3} & 1\bar{3}3\bar{1} \\
1 & 1\bar{3}3\bar{1} \\
\hline
 & 1\bar{3}08\bar{6}\bar{6}80\bar{3}1 \\
\hline
 & 7073|47971 \\
 & 2|1220 \\
 & 2829 \\
 & 70 \\
 & 1 \\
\hline
 & 24120
\end{array}
\left.\vphantom{\begin{array}{l}1\\2\\7\\1\end{array}}\right\}\text{Subtract.}
$$

```
'3'3'0'3'4'1'0'1 '↑ =·7 0 7 1 0 6 7 7
                  =
            '3 4 6 5 7 3 5 9
```

Ex. 3. Reduce '7'6'4'9'8'3'1'8 ₃↕₁ 6,8,10,15,6,5,6,7, to a dual logarithm less than 34657359 and to a dual number,

each digit of which not to exceed 5; also find the corresponding natural number.

$$'7'6'4'9'8'3'1'8\ {}_{8}\!\updownarrow_{2}\ 6,8,10,15,\bar{6}\,5,6,7,$$
$$49 \qquad\qquad 4\ \ 9\ \ 4\ \ 9$$
$$\|$$
$$'7'6'0'0'8'3'1'8\ {}_{8}\!\updownarrow_{2}\ 6,8,\ 6,\ 6,\bar{6},1.5\ 8,$$
$$6 \qquad\qquad 6\ \ \ 6\ \ \ 6$$
$$\|$$
$$'7'0'0'0'8'3'1'8\ {}_{8}\!\updownarrow_{2}\ 6,2,\ 6,\ 0,\bar{6},1,5,2,='13998974^{8}\uparrow^{8}_{8}$$
$$6\,\bar{1}\,\bar{5}\,\bar{2}$$
$$\overline{14166}$$

REDUCTION.

$$'7\ '\!\uparrow\ ='73760682 \qquad \downarrow,6,=57186108,$$
$$59775874, \qquad \overset{2}{\downarrow},2,=\ \ 1990066,$$
$$\overline{13984808} \qquad \overset{8}{\downarrow},6,=\ \ \ 599700,$$
$$14166 \qquad \overline{59775874,}$$
$$\overline{'13998974}$$

$$\downarrow,\ 10^{8}=690775528,$$
$$\downarrow,\ 2^{2}\ =138629436,$$
$$\overline{829404964,}$$
$$'13998974$$
$$\text{Dual log.}\ \ \overline{815405990,}$$

REDUCTION TO A COMMON NUMBER.

$$'1\ 3\ 9\ 9\ 8\ 9\ 7\ 4$$
$$'1\ 0\ 5\ 3\ 6\ 0\ 5\ 2$$
$$\overline{'3\ 4\ 6\ 2\ 9\ 2\ 2}$$
$$'3\ 0\ 1\ 5\ 1\ 0\ 2$$
$$\overline{'4\ 4\ 7\ 8\ 2\ 0}$$
$$'5\ 0\ 0\ 2\ 5\ 0$$
$$\overline{5,2,4,3,0,}$$

$$815405990,$$
$$\|$$
$$'1'3'5'0'0'0'0'0\ {}_{8}\!\overset{'}{\updownarrow}_{2}\ 0,0,0,5,2,4,3,0,$$
$$=$$
$$3477{\cdot}468588$$

To find the Corresponding Number.
Work unabridged.

$'3_2\uparrow$ $\quad 10\bar{3}030\bar{1}$

970|29900|0+

4|851|49|5−

$'5_3\uparrow$ $\quad$ 970|3+

1|0−

$'1\uparrow$ $\quad$ 965457198

965457 20

8689|1147|8

4|34456 $\quad \overset{4}{\downarrow}5$

8|7

86934|6021

...†1|7387 $\quad \overset{5}{\downarrow}2,$

3478 $\quad \overset{5}{\downarrow}4,$

261 $\quad \overset{7}{\downarrow}3,$

869·367147

twice 1738·734294

4 times 3477·468588

Ex. 4. Reduce $_5\!\downarrow_1$ 11,8,7,3,4,5,6,8, to a dual number o the lowest form, and then to a natural number.

$_5\!\downarrow_1$ 11,8,7,3,4,5,6,8

5540350 = 11,08,07,0 × 5

113194218

341198 = 31018 × 11

264 = 33 × "

113535680,

$\downarrow, 2^2 =$ 138629436,

252165116,

230258509,

21906607,

||

'0'0'1'4'0'4'7'8 $\updownarrow$ 2,3,

||

1·23258763

1·23258763

half 616293·815 natural number required

$\|$

$^{5}\!\downarrow^{1}$ 11,8,7,3,4,5,6,8, given dual number

$=$

1334142943, dual logarithm.

☞ 89. We have now arrived at these important conclusions, namely, that with the dual logarithms of 10^m and $(1+1)^n$, ($\downarrow$, 10· $\downarrow$, 2·), and a logarithm, numerically, not greater than 34657359, or '34657359 the dual logarithms of all the natural numbers between

$$+\infty \text{ and } 0$$

are instantly determined under the form (A); (m and n being whole numbers, positive or negative). The corresponding dual number may be represented thus,

$$v_1v_2v_3v_4v_5v_6v_7v_8 \; {}^{m}\!\updownarrow^{n} \; u_1u_2u_3u_4u_5u_6u_7u_8 \qquad \text{(A).}$$

The commas are omitted as unnecessary, the numerals 1; 2; 3; .... being employed to designate the positions of the dual digits v_1; v_2; v_3; ... &c.

It is not necessary that either v_1 or u_1 should exceed '3 or 3, and at least half these digits may be ciphers, hence, under this form (A) may be reduced, with great ease, to a natural number.

Therefore to determine in a direct manner the natural number corresponding to a dual logarithm requires but little numerical labour, since

(A) may assume the form (B) or (C), &c.

$$'0v_2'0v_4'0'0'0'0 \; {}^{m}\!\updownarrow^{n} \; u_10,u_30,u_5,u_6,u_7,u_8 \qquad \text{(B).}$$

$$'0v_2'0'0'v_5'v_6'v_7'v_8 \; {}^{m}\!\updownarrow^{n} \; u_10,u_3u_40,0,0,0, \qquad \text{(C).}$$

So that if the positions $u_5u_6u_7u_8$ be occupied, $v_5v_6v_7v_8$ become ciphers, and *vice versâ*. To have each of the last four digits not greater than 5, or '5 is of no moment.

These observations have been fully illustrated (87) *Ex.* 1 to 3, and (88) *Ex.* 1 to 4.

The solution of the converse problem, that is, to find the dual logarithm of any given natural number between

$$+\infty \text{ and } 0,$$

requires no additional labour or skill, since a natural number may be operated upon by 10· and 2· so as to give results either less than 1·41421356 or greater than ·70710678; and because

$$\begin{array}{ccc} 1{\cdot}41421356 & \quad\text{and}\quad & {\cdot}70710678 \\ \| & & \| \\ '0'0'0'0'5'8'4'3 \updownarrow, 3,6,1,0,0,0,0,0, & & '3'3'0'3'4'1'0'1\ '\uparrow \\ = & & = \\ 34657359, & & '34657359 \end{array}$$

Therefore any given common number may be reduced to a dual number of the form (A), (B), or (C), which dual number is easily converted to a dual logarithm. Some examples will make clear and illustrate these observations.

EXAMPLES.

Ex. 1. Find the dual number of the lowest terms, and also the dual logarithm of the common number 477735·784.

$$\begin{array}{r} 2)477735{\cdot}784 \\ \hline 2)238867\,892 \\ \hline \end{array}$$

$$\text{given No.} = 1{\cdot}19433\,946 \times 10^5 \times 2^2$$

$$\begin{array}{c} 1{\cdot}19433946 \times 10^5 \times 2^2 \\ \| \\ '0'1'3'0'0'0'0'0\ {}_5\!\updownarrow\!{}_2,\ 2,0,0,0,2,4,7,5, \text{ dual number.} \\ = \\ 5\downarrow, 10^{\cdot} + 2\downarrow, 2^{\cdot} + 17759327 \\ = \\ 1307681308, \text{ dual logarithm.} \end{array}$$

Ex. 2. Find the dual number of the lowest terms, and also the dual logarithm corresponding to the common number 1865·65413

$$\text{Given number} = \cdot 932827065 \times 10^3 \times 2$$

$$\|$$

$$\text{Dual number} = {}'1'0'0'0'2'2'9'3 \; {}^{3}\!\updownarrow^{1} \; 0,3,6,0,0,0,0,0,$$

$$\|$$

$$
\begin{array}{rl}
 & '6953546 \\
\downarrow, 2^{\cdot} = & 69314718, \\
\hline
 & 62361172, \\
3\downarrow, 10^{\cdot} = & 960775527, \\
\hline
 & 1023136699, = \text{dual log.}
\end{array}
$$

1865·65413 = ${}^{3}\!\downarrow 6,5,2,$ also, and may be expressed by a great variety of dual numbers, but the one previously found of the lowest terms requires but little calculation to convert it into either a logarithm or natural number.

A brief inspection of the numbers and examples exhibited in the subjoined concatenated arrangement will exemplify these important relations.

	I.	$\boxed{100000000}$	
Natural numbers situated between I. and II. may be reduced to a dual number by commencing with		$1 = \downarrow 1,$	
		$121 = \downarrow 2,$	
	or,	$1331 = \downarrow 3,$	
	II.	$\boxed{141421356}$	
Natural numbers between II. and III., divided by 2, may be reduced to a dual number by commencing with		$729 = {}'3 \uparrow$	
		$81 = {}'2 \uparrow$	
	or,	$9 = {}'1 \uparrow$	
	III.	$\boxed{200000000}$	
Natural numbers intervening between III. and IV., divided by 2, may commence with		$11 = \downarrow 1,$	
		$121 = \downarrow 2,$	
	or,	$1331 = \downarrow 3,$	

Example.

$$1{\cdot}32898724 = {}'0'0'2'0'1'2'3'3 \updownarrow 3,0,0,5, \; 28441721$$

Example.

$$1670{\cdot}74 = {}'2'0'0'0'1\,{}^{3}\!\updownarrow^{1} 0,3,1, \; '17988055$$

Example.

$$239{\cdot}468438 = {}'0'1'0'5\,{}^{2}\!\updownarrow^{1} 2,0,0,0,3\;4,2,6, \; 18010428,$$

IV. $\boxed{266000000}$

Natural numbers intervening between IV. and V., multiplied by 4, may commence with

$11 = \downarrow 1,$

$121 = \downarrow 2,$

or, $1331 = \downarrow 3,$

Example.

$$2797{\cdot}01465$$
$$=$$
$$'0'0'3 \, {}_4\!\updownarrow^{2} 1,2,0,5,0,2,5,7,$$
$$11271191,$$

V. $\boxed{345000000}$

Natural numbers between V. and VI., multiplied by 2, may be reduced to a dual number commencing with

$729 = '3 \uparrow$

$81 = '2 \uparrow$

or, $9 = '1 \uparrow$

Example.

$$3605$$
$$=$$
$$'3'1'1'0'0'3'7'6 \, {}_4\!\updownarrow^{\bar{1}} 0,0,0,0,2,$$
$$'32711615$$

VI. $\boxed{499999999}$

For natural numbers situated between VI. and VII., multiply by 2,

$11 = \downarrow 1,$

$121 = \downarrow 2,$

or, $1331 = \downarrow 3,$

Example.

$$\cdot 054549625$$
$$=$$
$$'0'1'0'2'0'1'0'0 \, \bar{1}\!\updownarrow^{\bar{1}} 1,0,2,0,3,0,0,0,$$
$$9733918,$$

VII. $\boxed{707106780}$

Natural numbers situated between VII. and VIII. may be reduced to a dual number commencing with

$$\left.\begin{array}{ll} 729 & ='3\uparrow \\ 81 & ='2\uparrow \\ \text{or, } 9 & ='1\uparrow \end{array}\right.$$

VIII. $\boxed{999999999}$

Example.

819·672683

=

'2'0'0'1 2 $\updownarrow$ 0,1,2,0,2,1,5,2,

'18885019

In the reduction of natural numbers to dual numbers by the simplest and easiest method, it may be observed that the natural numbers situated between VII. and VIII., and between I. and II., have neither to be multiplied nor divided by 2, and that all natural numbers may be reduced to dual numbers commencing with the dual digits $\downarrow 1$, $\downarrow 2$, or $\downarrow 3$, except those found between II. and III., and VII. and VIII., which, however, range no higher than '1$\uparrow$; '2$\uparrow$; or '3$\uparrow$.

CHAPTER VI.

PRACTICAL APPLICATION OF DUAL LOGARITHMS.

90. WITHOUT the use of tables, in a variety of ways, and under different circumstances, we have shown by easy, independent, and direct processes, how any two of the three corresponding numbers

(NATURAL NUMBER); (DUAL NUMBER); (DUAL LOGARITHM);

might be found, the remaining one being given.

Any one of these convertible numbers being given, the other two may be also found by employing Tables I. and II. Table I. is of the ascending branch, in which the natural numbers range from

1· to 2·99161136.

Table II. is of the descending branch, of which the natural numbers range from

·29916114 to 1.

Hence, when these tables are employed, the powers of the base $1+1$ or 2 are dispensed with, and only the powers of 10 retained. When operations are performed with dual numbers in their lowest terms, and tables used, it is not necessary that such tables should range beyond the natural numbers

from 1·41421356 to 1. and from 1. to ·70710678; (89).

Those who operate with dual logarithms should remember (91), (92), (93).

☞ 91. In future, when one of the three corresponding numbers, *Natural number*, *Dual number*, *Dual Logarithm*, is known, either of the other two is set down and marked (91), without stating whether obtained through inspecting the tables or found by direct calculation.

☞ 92. Natural numbers are prepared for logarithmic operations, when the tables are consulted, by simply changing the decimal point one, two, three, &c. places to the right or left until the natural numbers are to be found between

1·000 ...... and 2·9916113612 .. ;
or between
·299161136 ..... and ·99999999 ....

The results obtained must be pointed off accordingly, because (18) the changing of the decimal point one, two, three, &c. places to the right or left being tantamount to multiplying or dividing by 10, 100, 1000, &c.

☞ 93. Dual logarithms are whole numbers; those of the ascending branch have a comma to the right, below, while those of the descending branch are designated by a comma placed above to the left (33), (38). Thus the dual logarithm of 2· as well as the dual logarithm of $\frac{1}{2}$ is the whole number 69314718 but written

$$\downarrow, (2\cdot) = 69314718,$$
$$\downarrow, (\cdot 5) = {}'69314718$$

If the dual logarithms of the ascending branch be considered positive, those of the descending branch must be taken as negative, and *vice versâ* (47).

☞ Hereafter the term *logarithms* will be applied to designate dual logarithms, when the contrary is not specified. ☜

The consideration of a few special examples will aid the retention of the memory and pointedly exemplify the preceding directions (90), (91), (92), (93).

Examples.

Ex. 1. With what numbers must the tables be entered to find the logarithms of

245·672	98·3657	4·846321
2·345678*	1·345672*	33·4455
·0012345	·04763	·8765432*
·001	·1	100·
1000.	9473.	1276.

Answer.

The numbers marked (*) have the decimal point in the required position.

2·45672	(Tab. I.)	·983657	(Tab. II.)	·4846321	(Tab. II.)
1·2345	(Tab. I.)	·4763	(Tab. II.)	·334455	(Tab. II.)
1·	(Tab. I.)	1·	(Tab. I.)	1·	(Tab. I.)
1·	(Tab. I.)	·9473	(Tab. II.)	1·276	(Tab. I.)

Ex. 2. Required the sums of the dual logarithms

47634512,	and 34275652,	*Ans.*	81910164,
'12345678	and '56324217	*Ans.*	'68669895
'56324217	and 47634512,	*Ans.*	'8689705
'12345678	and 34275652,	*Ans.*	21929974,

47634512,	'12345678	Add together.
34275652,	'56324217	
81910164,	'68669895	

Add together { '56324217
47634512,
'8689705

The comma being with the difference on the side of the greater logarithm.

Add together { ,34275652,
'12345678
21929974,

Ex. 3. Add together 69314718, and '69314718

$\downarrow$, (2·) = 69314718,
$\downarrow$, (·5) = '69314718
$\downarrow$, (1·) = 00000000

94. When a logarithm has to be subtracted the distinguishing comma of such logarithm must be changed, or supposed to be changed, from left to right or from right to left, and then the logarithm is to be operated with as in addition. These directions are analagous to the rules laid down for addition and subtraction of algebra.

Ex. 4. Add together '34566542 and '2145635 and subtract 12332198,

'34566542
'2145635
12332198,
'49044375

Ex. 5. Add together 12332198, and '2145635 and subtract '34566542

12332198,
'2145635
10186563,
34566542,
44753105,

Ex. 6. From 12332198, take '34566542

12332198,
'34566542
―――――
46898740,

Ex. 7. How many places of figures must the decimal point be removed to the right or left in the continued product of the natural numbers (A), to give the continued product of the natural numbers (B).

(A).					(B).
·56789	point removed		3, to right	becomes	576·89
1·2345	"	"	'2 to left	"	·012345
2·0476	"	"	1, to right	"	20·476
1·9834	"	"	4, to right	"	19834·
·6666	"	"	0	"	·6666
2·345	"	"	0	"	2·345
1·000	"	"	'4 to left	"	·0001
			2,		

*** If the decimal point be moved two places to the right, the continued product of (A) is altered to the continued product of (B) (91).

Ex. 8. How many places of figures must the decimal point be removed to the right or left in the continued product of 365·1768; 7454·83; 1650·22; ·000820 divided by the continued product of 18·636; 83·344; ·22222

Multiply		Divide by	
·3651768	3,	1·8636	1,
·745483	4,	·83344	2,
1·65022	3,	2·2222	'1
·820	'3		2,
	7,		
	'2		
	5,		

The decimal point has to be moved five places to the right (92).

95. The comma with the number found for the factors of the divisor is changed from right to left or from left to right before being incorporated with the number obtained for the factors of the dividend. Thus, in *Ex.* 8. 2, is changed to '2; then 7, and '2 together make 5, With a *little experience*, a glance at the numbers to be incorpo-

rated and at those whose logarithms are found will be sufficient to decide the position of the decimal point, without entering into any formal method of calculation.

MULTIPLICATION BY LOGARITHMS.

RULE.

96. Add the logarithms of the factors together; then the natural number answering to the sum will be the product required.

Observing, in the addition, that if the logarithms of the ascending branch be considered affirmative, those of the descending branch must be taken as negative, after the manner of positive and negative quantities in algebra (93).

EXAMPLES.

Ex. 1. Multiply 953·77426 and 12638·1188 together by logarithms.

$$\begin{array}{lll} 4, & \downarrow, (1{\cdot}26381188) = & 23413252, \\ \underline{3,} & \downarrow, ({\cdot}95377426) = & \underline{{}'4732831} \\ 7, & & 18680421, = \downarrow, (1{\cdot}20539124) \end{array}$$

∴ Product=12053912·4 which is true to the last figure.

Ex. 2. Multiply 90·986868; 1943·34858 and 295·429627 continually together.

$$\begin{array}{lll} 2, & \downarrow, (2{\cdot}95429627) = & 108326047, \\ 1, & \downarrow, (1{\cdot}94334858) = & 66441255, \\ \underline{2,} & \downarrow, ({\cdot}90986868) = & \underline{{}'9445506} \\ 5, & & 165321796, \\ & \downarrow, (\tfrac{1}{10}) = & \underline{{}'230258509} \\ & & {}'64936713 = \downarrow, ({\cdot}52237607) \end{array}$$

∴ Product=522376·07.

97. ☞ When some quantities have to be added and others subtracted to produce a *result* it is contradictory to call such *result* a *sum* or *difference;* therefore, for the sake of order and distinctness, hereafter a *result* so obtained will be called the AMOUNT and not the *sum* or *difference;* further, the misuse of the terms *add* and *subtract,* so well defined in common arithmetic, will be avoided in such cases by using the term COLLECT.

In *Ex.* 2 the *amount* 165321796, exceeds 109581215, the

limit of Table I., but when combined with $\downarrow, (\frac{1}{10}) =$ '230258509, as directed (86), the amount '64936713 is found in Table II. The limit of the logarithms of Table II. being '120677297.

The rule (96) for multiplication may now be written as follows.

RULE.

Collect the *amount* of the logarithms of the factors to be multiplied; the natural number answering to this amount will be the product required.

Observing that in the *collecting* together into one *amount*, if the logarithms of the ascending branch be considered affirmative, those of the descending branch must be taken as negative.

Arithmetical complement.—Begin at the left, set down minus one, written $\bar{1}$, *then take each of the figures from* 9 *except the last figure on the right, which must be taken from* 10.

98. The *amount* may be found by adding, and subtracting avoided, if the arithmetical complements of that class whose sum is numerically least, be substituted for the numbers themselves. It requires but a trifling inspection to decide which of the two classes of numbers has the greater numerical sum.

See Example 1.

$$\begin{array}{l} 23413252, \\ \bar{1}5267169 \text{ ar. co.} \\ \hline 18680421, = \downarrow, (1{\cdot}20539124) \end{array}$$

See Example 2.

$$\begin{array}{r} \bar{1}891673953 \text{ ar. co.} \\ \bar{1}33558745 \text{ ar. co.} \\ {'}9445506 \phantom{\text{ ar. co.}} \\ {'}230258509 \phantom{\text{ ar. co.}} \\ \hline {'}0064936713; \phantom{\text{ ar. co.}} \end{array}$$

or='64936713 since a whole number is not altered in value by prefixing ciphers to the left. In such cases no allowance has to be made on account of having to employ arithmetical complements, which is one of the advantages of this over any other system. The manage-

ment of common logarithms is rendered difficult because the decimal part is always taken as positive, while the whole numbers or indices may be either positive or negative. The common logarithm of ·00012345 is made up of two parts -4 and $+$·0914911 written $\bar{4}.0914911$.

Ex. 3. Multiply ·00285095 × 82·550825 × ·0092730306 by logarithms.

'3	$\downarrow$, (2·85095)	= 104765225,
'2	$\downarrow$, (·92730306)	= '7547488
2,	$\downarrow$, (·82550825)	= '19175607
'3		78042130, = $\downarrow$, (2·18239151)

∴ Product = ·00218239151 (91) (92) (93)

with the arithmetical complements of the logarithms.

$$\left.\begin{array}{r} 104765225, \\ \bar{1}2452512 \\ \bar{1}80824393 \end{array}\right\} \text{Add.}$$

$\downarrow$, (2·18239151) = 78042130, (97) (98).

Ex. 4. Multiply ·159037822 × ·2778189 × ·00290188 by logarithms.

'1	$\downarrow$, (1·59038822)	= 46397186,
'1	$\downarrow$, (2·778189)	= 102179928,
'3	$\downarrow$, (2·90188)	= 106535881,
2,	$\downarrow$, (·1)	= $\bar{1}769741491$ = ar. co. '230258509
'3	$\downarrow$, (1·2821783)	= 24854486,

∴ Product = ·0012821783 (91) (92) (94) (98).

Examples for Practice.

Ex. 5. Multiply 4·0763 by 9·8432 by logarithms.

Ans. 40·12383.

Ex. 6. Multiply 2876·9; ·10674; ·098762; and

Division by Logarithms.

Rule.

99. If the comma appended to the logarithm of the divisor be on the right, remove it to the left; if on the left, remove it to the right; then find the *amount* of the logarithms of the dividend and divisor, the number answering to this amount will be the quotient required.

EXAMPLES.

Ex. 1. Divide 4640·91 by 266·445347.

Ans. 17·417868.

$$
\begin{array}{lrl}
4, & \downarrow, (\cdot 464091) & = \bar{1}23232537 \\
\underline{2,} & \downarrow, (2\cdot 66445347) & = \bar{1}02000104 \\
2, & & \\
\underline{'1} & & \\
1, & & \underline{230258509}, \downarrow, (10\cdot) \\
 & \downarrow, (1\cdot 7417868) & = 55491150,
\end{array}
$$

∴ Quotient = 17·417868 (94) (97) (98).

Ex. 2. Divide 46·4091 by 2664·45347

$$
\begin{array}{lrl}
 & 2, \quad \downarrow, (\cdot 464091) & = \bar{1}23232537 \\
\text{Diff.} & \frac{3,}{'1} \quad \downarrow, (2\cdot 66445347) & = \bar{1}02000104 \\
 & & \underline{230258509}, \downarrow, (10\cdot) \\
\text{For } \downarrow, (10\cdot) & \frac{'1}{'2} \quad \therefore \downarrow, (1\cdot 7417868) & = 55491150,
\end{array}
$$

∴ Quotient = ·017417868 (92) (94) (98).

Ex. 3. Divide 464091· by ·00266445347

$$
\begin{array}{lrl}
 & 6, \quad \downarrow, (\cdot 464091) & = \bar{1}23232537 \\
\text{Diff.} & \frac{'3}{9,} \quad \downarrow, (2\cdot 66445347) & = \bar{1}02000104 \\
 & & \underline{230258509}, \downarrow, (10\cdot) \\
\text{For } \downarrow, (10\cdot) & \frac{'1}{8,} \quad \downarrow, (1\cdot 7417868) & = 55491150,
\end{array}
$$

∴ Quotient = 174178680.

Ex. 4. Required the value of $1174 \dfrac{\cdot 744438}{\cdot 8197015}$

$\downarrow, (1\cdot 174) = 16041673$, $\downarrow, (\cdot 744438) = '29512575$

$\downarrow, (\cdot 8197015) = '19881508$;

$$
\begin{array}{lrr}
\text{Then,} & 16041673, & 3, \\
 & \bar{1}70487425 & 0 \\
 & \underline{19881508,} & \underline{0} \\
\downarrow, (1\cdot 06620547) = & 6410606, & 3,
\end{array}
$$

∴ 1066·20547 = the required value.

EXAMPLES FOR PRACTICE.

Ex. 5. Divide ·0678593 by 1234·593 by logarithms.
Ans. ·0000549648.

Ex. 6. Divide 19956·7 by ·048235 by logarithms.
Ans. ·413739.

Ex. 7. Divide 10·23674 by 4·96523 by logarithms.
Ans. 2·061685.

THE RULE OF THREE BY LOGARITHMS.

RULE.

100. Add the logarithms of the second and third terms together, and subtract the logarithm of the first from their sum, then the natural number answering to the *amount*, found according to the foregoing rules, will be the fourth term required.

EXAMPLES.

Ex. 1. Find a fourth proportional to 37·516227; 14·7732974 and 135·239606.

$\downarrow$, (·37516227) = '98039667; $\downarrow$, (1·47732974) = 39023624,
$\downarrow$, (1·35239606) = 30187789,

1,	39023624,
$\frac{2,}{3,}$	30187789,
$\frac{'2}{1,}$	98039667,
	167251080,
$\frac{1,}{2,}$	'230258509
	'63007429

$\Vert$

'6 $\updownarrow$ 0,0,2,0,8,9,8,0,

=

·53255223 (85) (86) (97) (98).

Therefore 53·255223 is the fourth proportional required.

Ex. 2. Find a fourth proportional to ·056722234; ·71882751 and ·34728582 by logarithms.

$\downarrow$, (·56722234) = '56700394; $\downarrow$, (·71882751) = '33013389
and $\downarrow$, (·34728582) = '105760716

These logarithms may be set down in the following order

0	'33013389
0	'105760716
1,	$\bar{1}$43299606
1,	'82073711

$$\|$$

'7'8'3 '$\updownarrow$ 0,0,0,1,9,0,7,2,

$$=$$

·44010713 (85) (86)

∴ 4·4010713 = the fourth proportional.

Ex. 3. Find the interest of £279. 5*s*. for 274 days at $4\frac{1}{2}$ per cent. per annum.

2,	$\downarrow$, (1·00)	000000000
3,	$\downarrow$, (·365)	100785793, with comma moved.
2,	$\downarrow$, (2·7925)	102693726,
2,	$\downarrow$, (2·74)	100795790,
1,	$\downarrow$, (·45)	$\bar{1}$20149231 ar. co.
		224424540,
	$\downarrow$, (10·)	'230258509
		'5833969

$$\|$$

'0'5'8'0'8'3'9'9 '$\uparrow$

$$=$$

·94332951

Ans. £9·4332951.

EXAMPLES FOR PRACTICE.

Ex. 4. Find a fourth proportional to 12·678; 14·065 and 100·979. *Ans.* 112·0263 nearly.

Ex. 5. Find a fourth proportional to 1·9864; ·4678 and 50·4567. *Ans.* 11·88262 nearly.

Ex. 6. Find a fourth proportional to ·09658; ·24958 and ·008967. *Ans.* ·02317234 nearly.

Ex. 7. Find a mean proportional between ·498621 and 2·9587. *Ans.* 17·55623 nearly.

Involution, or the Raising of Powers.

Rule.

101. Find the logarithm of the given number and multiply it by the index of the proposed power; the natural number answering to the result will be the power required.

Examples.

Ex. 1. Find the square of 2·75606318.

$\downarrow$, (2·75606318) = 101380328,
2 for the square.

202760656,
1, $\downarrow$, (·1) '230258509

$\downarrow$, (·75957849) = '27497853

∴ Square = 7·5957849 = '2'6'4 1 $\overset{5}{\updownarrow}$ 4,6,5,4,

Ex. 2. Find the cube of 7·2536238 by dual logarithms.

1, $\downarrow$, (·72536238) = '32108405
3 3 for the cube.

3, $\downarrow$, (·38164972) = '963252l5

∴ Cube = 381·64972 = 9'1'5 $\overset{5}{\updownarrow}$ 3 4,5,3,3,

Ex. 3. Find the fourth power of 7·64926.

1, $\downarrow$, (·764926) = '2 6 7 9 7 6 2 3
4 4 for the 4th power.

4, $\downarrow$, (·34235572) = '1 0 7 1 9 0 4 9 2
'1 0 7 1 6 5 9 5 0

'0'0'0'2'4'5'4'2

∴ 4th power = 3423·5572 = '10'1'8'2'4'5'4'2 $\uparrow$ 4

Ex. 5. Find the 5th power of ·86108347.

0, $\downarrow$, (·86108347) = '14956388
5 5 for the 5th power.

0, $\downarrow$, (·47339771) = '74781940

∴ 5th power = ·47339771

Ex. 6. Find the 365th power of 1·00621623 by dual logarithms.

$$
\begin{array}{r}
\downarrow, (1\cdot00621623)=619700,\\
365\\
\hline
3098500\\
37182\\
18591\\
\hline
\end{array}
$$

$\downarrow, (9\cdot60136224)=226190500, = 8\downarrow, 1,8,7,5,5,4,1,4,$

EXAMPLES FOR PRACTICE.

Ex. 7. Required the square of 6·05987 and the cube of ·176546. *Ans.* 36·72203 and ·005502674.

Ex. 8. Required the 4th power of ·076543 and the 7th power of 1·09684. *Ans.* ·0000343259 and 1·909864.

EVOLUTION,
OR THE EXTRACTION OF ROOTS BY DUAL LOGARITHMS.

RULE.

102. Find the logarithm of the given number, and divide it by 2 for the square root, 3 for the cube root, &c., and the natural number answering to the result will be the root required.

If the root be expressed by a fraction, multiply the logarithm of the given number by the numerator of the index, and divide the product by the denominator, for the logarithm of the root so expressed.

EXAMPLES.

Ex. 1. Find the square root of 1579·15522.

$2)\underline{3,}$ $\downarrow, (1\cdot57915522)=$ 45689003,
1 and 1 over $\downarrow, (10^{\cdot})=230258509,$

For the square root 2)275947512,

$\downarrow, (3\cdot9738586)=137973756, ='0'0'6'5'5'3'8'0 \uparrow 4$

∴ Square root = 39·738586.

Ex. 2. Required the cube root of 35·64188.

$3)\underline{2,}$ $\downarrow, (\cdot3564188)='103164887$
0 and 2 over $\downarrow, (10^{2})=$ 460517018,

For the cube root 3)357352131,

$\downarrow, (3\cdot2909414)=$ 119117377,

Ex. 3. Required the 5th root of 2·13768341.

$\downarrow$, (2·13768341) = $\underline{75972273}$,

Divide by 5 for the 5th root 15194455, = '1'1'3'8 $\overset{4}{\updownarrow}$ 1,5,7,

$\downarrow$, (1·16409567) = 15194455,

Ex. 4. Find the 365th root of 2·13768341.

365)$\underline{75972273}$, = $\downarrow$, (2·13768341)

$\downarrow$, (1·00208359) = 208143, = $\downarrow$, 0,0,2,0,8,2,4,3,

Ex. 5. Find the value of $(1810{\cdot}78553)^{\frac{2}{3}}$.

3, $\downarrow$, (1·81078553) = 59376074,
2   2

3)6,   3)118752148,

2, $\downarrow$, (1·48563228) = 39584049, = $\downarrow$ 4,1,4,6,5,1,4,4,

∴ 148·563228 is the required value.

Ex. 6. What is the value of $({\cdot}66665095)^{\frac{3}{4}}$.

$\downarrow$, (·66665095) = '40548877
3

4)'121646631

$\downarrow$, (·73777491) = '30411658

Ex. 7. Find the cube root of ·000213768314.

The decimal point has to be changed four places to the left in 2·13768314 to produce the given number; then 3)$\underline{'4}$
'1 and '1 over.

$\downarrow$, (2·13768314) = 75972273,
'230258509 for the '1 over.

For the cube root 3) '154286236

$\downarrow$, (·5979265) = '51428745 = '4'9'3 $\overset{4}{\updownarrow}$ 6,0,9,1,7

∴ Cube root = ·05979265

Ex. 8. Find the value of $(\cdot 0066665095)^{\frac{2}{7}}$ by logarithms.

The decimal point has to be removed two places to left in ·66665905 to produce the given number;

then '2

$$\begin{array}{r} 2 \\ \hline 7)\text{'}4 \\ \hline 0 \text{ and } \dfrac{\text{'}4}{7} \text{ over} \end{array}$$

But a moment's reflection will give such results without a formal calculation.

$$\begin{array}{rlr} & \downarrow, (\cdot 66665095) = & \text{'}40548877 \\ & \downarrow, (\cdot 01) \qquad\quad = & \text{'}460517018 \\ \hline \therefore & \downarrow, (\cdot 0066665095) = & \text{'}501065895 \\ & & 2 \\ \hline & & 7)\text{'}1002131790 \\ \hline & \downarrow, (\cdot 23892233) = & \text{'}143161684 \end{array}$$

REDUCTION.

'143161684

=

$\text{'}0\text{'}4\text{'}5\text{'}1\text{'}1\text{'}8\text{'}6\text{'}1 \uparrow \frac{1}{4} = \cdot 23892233$

EXAMPLES FOR PRACTICE.

Ex. 9. Required the square root of 365·5674 by logarithms. *Ans.* 19·11981 nearly.

Ex. 10. Required the cube root of 2·987635; the 4th root of ·967845; and the 7th root of ·098674.

Ans. 1·440265; ·9918624 and ·7183146.

Ex. 11. Required the value of $\left(\frac{112}{1727}\right)^{\frac{2}{3}}$

Ans. ·1937115.

CHAPTER VII.

APPLICATION OF DUAL LOGARITHMS TO THE SOLUTION OF MISCELLANEOUS IMPORTANT PROBLEMS.

Ex. 1. How much would £1 amount to in 52 years at 5 per cent. compound interest? *Ans.* £12·6428054.

$$\begin{array}{c} 1{\cdot}05 \\ \| \\ \text{'0'0'1'0'0'1'0'1} \updownarrow \text{0,5,0,0,4,0,0,2,} \\ = \\ 4879016, \\ 4879016, \times 52 = 253708832, \\ \| \\ \downarrow, (12{\cdot}6428054) = 10 \downarrow, 2,4,4,0,8,3,5,5, \end{array}$$

£20 at the same rate and for the same time would amount to £252·856108 = 20 × 12·6428054.

Ex. 2. If £20 amount to £252·856108 in 52 years, compound interest, what is the rate per cent. per annum? *Ans.* 5 per cent.

$$\frac{252{\cdot}856108}{20} = 12{\cdot}6428054$$

$$\frac{\downarrow, (12{\cdot}6428054)}{52} = 4879016, = \downarrow, (1{\cdot}05).$$

Ex. 3. In how many years will £20 amount to £252·856108 at 5 per cent. compound interest? *Ans.* 52 years.

$$\frac{252{\cdot}856108}{20} = 12{\cdot}6428054$$

$$\frac{\downarrow, (12{\cdot}6428054)}{\downarrow, (1{\cdot}05)} = 52 \text{ years.}$$

Ex. 4. In what time will £1 amount to £2, or in other words, in what time will a sum of money double itself at 5 per cent. compound interest? *Ans.* 14·2067 years.

$$\frac{\downarrow, (2{\cdot})}{\downarrow, (1{\cdot}05)} = \frac{69314718,}{4879016,} = 14{\cdot}2067.$$

Ex. 5. In what time will £1 amount to £1035256190, the national debt of England, at 5 per cent. compound interest? *Ans.* 425·453 years.

$$\frac{1035256190,}{1\cdot} = 1035256190,$$

$$\frac{\downarrow, (1035256190\cdot)}{\downarrow, (1\cdot 05)} = \frac{2075791483,}{4879016,} = 425\cdot 453 \text{ yrs.}$$

Ex. 6. What will £200 amount to in 25 years at $6\frac{1}{2}$ per cent. compound interest, supposing the interest to be receivable half-yearly? *Ans.* £989·766784.

It is evident that the amount will be the same as £200 for 50 years at $3\frac{1}{4}$ per cent.

$$\begin{aligned} \downarrow, (1\cdot 0325) &= 3198304, \\ &\;\; 50 \\ \downarrow, (989\cdot 766784) = 4 \downarrow, 22233762 &= 159915200, \end{aligned}$$

These solutions will serve as models to show how all similar questions of compound interest may be solved; the principle, amount, time, and rate of interest, being altered to suit each particular example.

103. The circumference of a circle whose diameter $=1$ is generally represented by $\pi = 3\cdot 14159265358979$ nearly.

$$3\cdot 14159265358979$$
$$=$$
$$\frac{10}{3} \text{ of}$$
$$'0'6'0'0'0'0'4'0'3'4'0'0'0'0'0'0 \updownarrow, \begin{array}{l} 0,0,1,0,6,0,0,0,0,0,1, \\ \quad 1,4,0,7,2,8, \end{array}$$
$$=$$
$$'59242918476535 83$$
$$\text{ar. co.} \quad \bar{1}4075708152346417$$

$$\frac{10}{3} = \frac{1\cdot}{\cdot 3} = \frac{3\cdot}{\cdot 9}$$

$$\begin{aligned} \downarrow, (3) &= 1098612288668 10969, \\ \text{minus } '1\,' \uparrow \quad &= 1053605156578 2630, \\ &\; \bar{1}4075708152346417 \\ \hline \therefore\ \downarrow, (\pi) &= 1144729885849 40016, \end{aligned}$$

RULE.

Multiply the diameter of a circle by $\frac{10}{3}$ and $'6 \overset{3}{\underset{2}{\updownarrow}} 1,0,6,$ and the product will be the circumference nearly.

Ex. 7. The diameter of a circle is 34 feet; what is the length of the circumference?

$$
\begin{array}{r}
3)\underline{3\;4\;0\cdot} \\
1\;1\,|\,3\cdot3\,|\,3\;3\,|\,3\;3\,+ \\
|\,6\;8\,|\,0\;0\,|\,0\;0\,- \\
1\,|\,7\;0\,|\,0\;0\,+ \\
2\,|\,2\;7\,- \\
2\,+ \\
\hline
1\;0\;6\;7\;0\;1\;0\;8 \\
1\;0\;6\;7\;0 \\
\hline
1\;0\;6\;8\;0\,|\,7\;7\;8 \\
|\,6\;4\;1 \\
\hline
\end{array}
\qquad 6\underset{2}{\uparrow}
$$

$$\text{Cicumference} = 1\;0\;6\cdot8\;1\;4\;1\;9$$

Ex. 8. Find the length of an arc of a circle of $22^\circ\ 29'\ 29''\cdot28$; radius$=1$.

$$\text{Length of } 180^\circ = 3\cdot14159265\ldots$$

$$\therefore\ 180^\circ : 3\cdot14159265 :: 22^\circ\ 29'\ 29''\cdot28 : \cdot39255015$$

$$
\begin{array}{lll}
 & 60)\underline{29''\cdot28} & \\
60)\underline{29'} & \cdot488 & \\
22^\circ & \cdot4914666 & = 10\times2(1\cdot12457333) \\
 & & = 10\times2\downarrow 11740371,
\end{array}
$$

104. Reduction of the ratio of

$$
\begin{array}{rcl}
180 : 3\cdot14159265 = 10^2\times\cdot9\times2 & : & '5924292\uparrow\frac{10}{3} \\
= \quad 1 & : & \dfrac{'5924292\uparrow}{'1\uparrow}\times\dfrac{1}{10\times6} \\
= \quad 1 & : & \dfrac{'5924292\uparrow}{'105360052\uparrow}\times\dfrac{1}{60} \\
= \quad 1 & : & \frac{1}{60}\downarrow\ 4611760,
\end{array}
$$

This ratio being constant may be applied in other cases.

$$
\begin{array}{rl}
\text{Mult. } 10\times2\downarrow & 11740371, \\
\text{by}\quad \frac{1}{60}\downarrow & \underline{4611760,} \\
\cdot39255015 = \frac{1}{3}\downarrow & 16352131, = \frac{1}{3}(1\cdot17765044).
\end{array}
$$

Ex. 9. Find the length of an arc of a circle of $95° \ 34' \ 48''·96$ radius = 1. *Ans.* 1·6681902.

$$\begin{array}{l} 60)\underline{48''·96} \\ 60)\underline{34' \quad ·816} \\ 95° \quad ·5802666 = 10^2(·955802666) \end{array}$$

$\downarrow$, (·955802666) = '4520386

Constant (*Ex.* 8.) 4611760,

$\downarrow$, (1·00091411) = 91374.

$$\frac{100}{60} = \frac{10}{6} \therefore \quad 6)\underline{10·0091411} \quad 1·6681902$$

Ex. 10. The diameter of the earth in latitude 45° is said to be 7896·2814 statute miles, what is the area of a circle of this diameter? *Ans.* 48906550· sq. miles.

105. If D be the diameter of a circle and $C = \frac{\pi}{4} =$ ·78539816, then the area

$$= D^2 \times C. \text{ or } \downarrow, (D^2) + \downarrow, (C) = \downarrow, (\text{area})$$

$$\begin{array}{lll} 4,\ldots & \therefore \downarrow, (·78962814) & = '23682471 \\ \underline{2} & & \underline{2} \\ 8, & \downarrow, (D^2) & = '47364942 \\ & \downarrow, (·78539816) & = '\underline{24156447} \\ & \downarrow, (·48906550) & = '71521389 \end{array}$$

Ex. 11. The angles of a right-angled triangle being given, to find three sides that will contain these angles.

106. In solving this problem

the numbers 1 3 6 10 15 21 28 36 ....
and also 2 6 12 20 30 42 56 ....
will be required, see tables (23) (49).

$$\frac{1}{(1-x)^3} = 1 + 3x + 6x^2 + 10x^3 + 15x^4 + \ldots.$$

Rule I.

To find the hypothenuse.

Set down 10000· times the length of the arc that measures the least of the acute angles and divide it by $\sqrt{2} = 1·4142136$, the quotient will be the dual logarithm of the hypothenuse.

Rule II.

To find the base and perpendicular.

Let h represent the dual logarithm of the hypothenuse, found by Rule I., and from the square of the h take h; then h^2-3h+2; h^2-5h+6; $h^2-7h+12$, &c. are found by merely subtracting $2h$ and adding at each step a term of the series 2 6 12 20....

In what follows $[h^2-h]$ is put for h^2-h divided by 10^8;
$[h^2-3h+2]$ for h^2-3h+2 divided by 10^8;
and so on.

Put A=the length of the arc to radius 1, and $B=[h^2-h]$

$$C=\frac{A}{3}[h^2-3h+2];\ D=\frac{B}{6}[h^2-5h+6];\ E=\frac{C}{10}[h^2-7h+12];$$

$$F=\frac{D}{15}[h^2-9h+20];\ \text{\&c.}$$

Then $A-C+E-G+I-$&c. gives the base and
Radius$-B+D-F+H-$&c. gives the perpendicular.

These rules are demonstrated in the author's work on the science of dual arithmetic applied to Trigonometry.

Ex. 12. Find the three sides of a right-angled triangle that will have one of its acute angles equal $21^\circ\ 19'\ 37''{\cdot}8$.

The length of an arc of $21^\circ\ 19'\ 37''{\cdot}8 = {\cdot}37222928$, radius$=1{\cdot}$

$$\frac{3722{\cdot}2928}{1{\cdot}4142136}=2632{\cdot}058=h;\ \text{and}\ h^2=6927732.$$

Then, according to Rule I. of the last example, 2632, is the dual logarithm of the hypothenuse. Therefore (85),

$$\downarrow,\ (1{\cdot}00002632)=2632,$$

$$\therefore\quad \text{Hypothenuse}=1{\cdot}00002632.$$

$$\begin{array}{rl}
6927732{\cdot} & \\
2632{\cdot} & \\
\hline
6925100{\cdot} & =h^2-h \\
5264{\cdot} & \\
\hline
6919836{\cdot} & =h^2-3h \\
5264{\cdot} & \\
\hline
6914572{\cdot} & =h^2-5h \\
\text{\&c.} & \quad \text{\&c.}
\end{array}$$

$$
\begin{aligned}
\text{Radius} &= 1\cdot00000000+\\
\text{Length of arc } A &= \cdot37222928+\\
B &= \cdot06925100- = [h^2-h]\\
C &= 858589- = [h^2-\ 3h+\ 2]\frac{A}{3}\\
D &= 79808+ = [h^2-\ 5h+\ 6]\frac{B}{6}\\
E &= 5930+ = [h^2-\ 7h+12]\frac{C}{10}\\
F &= 367- = [h^2-\ 9h+20]\frac{D}{15}\\
G &= 19- = [h^2-11h+30]\frac{E}{21}\\
H &= 1+ = [h^2-13h+42]\frac{F}{28}
\end{aligned}
$$

A	$37222928+$	Radius	$=1\cdot00000000+$
C	$\bar{1}141411$ ar. co.	B	$\bar{1}3074900$ ar. co.
E	$5930+$	D	$79808+$
G	$\bar{1}81$ ar. co.	F	$\bar{1}633$ ar. co.
Perpendicular	$=\cdot36370250$	H	$1+$
		Base	$=\cdot93154342$

$1\cdot00002632$; $\cdot93154342$ and $\cdot36370250$ are the sides of the required right-angled triangle. If necessary we may now find the sine and cosine of any angle as $21°\ 19'\ 37''\cdot8$ true to eight places of decimals;

$$\sin 21°\ 19'\ 37''\cdot8 = \frac{\cdot36370250}{1\cdot00002632} = \cdot36369293$$

Ex. 13. Given the three sides of a plane triangle respectively equal $7891\cdot23456$ (a); $12345\cdot67891$ (b); and $8912\cdot34567$ (c) to find the area.

Put $2s=a+b+c$. Then it is well known that the area is equal $\sqrt{s(s-a)(s-b)(s-c)}$; or

$$\downarrow, (\text{Area}) = \tfrac{1}{2}\left[\downarrow, s + \downarrow, (s-a) - \downarrow, (s-b) + \downarrow, (s-c)\right]$$

$s=4574\cdot62957$; $s-a=6683\cdot39501$; $s-b=2228\cdot95066$; $s-c=5662\cdot28390$

4,	$\downarrow,(s)$	37669724,
4,	$\downarrow,(s-a)$	$\bar{1}59704096$ ar. co.
3,	$\downarrow,(s-b)$	80153092,
4,	$\downarrow,(s-c)$	$\bar{1}43124223$ ar. co.
2)15,	$\downarrow,(10)$	230258509,
7 and 1 over		2)250909644,
		125454822, $=\downarrow,(3{\cdot}50625396)$

$\therefore$ Area $=35062539{\cdot}6$

See "Dual Arithmetic, a new Art," page 106.

107. If a musical string be stretched over two edges, and a force applied at the middle to draw the string from a straight position, then free the string, and the elasticity that resisted the force will operate to reform the straight position. In a similar manner as in the pendulum, the string has obtained a momentum when it reaches its straight position, which must be discharged before it can come to rest through a series of vibrations. The pitch depends on the number of vibrations in a unit of time, and the number of vibrations depend on the weight, length, and tension of the string. Suppose an edge placed at the middle of a string and one half of the length set vibrating, it will make twice as many vibrations as the whole string would do in a unit of time, and the different pitch of tones thus produced is called an *octave*. In the chromatic scale an octave is divided into twelve equal parts. Let L be the length of a string that will sound a given note, say C, and l the part of the same string and of the same tension that will sound any note above C.

Put m for the number of octaves above C, and n the position of the note in the next octave above the nth.

$$\text{Then} \quad l=\frac{L}{2^{(m+\frac{n}{12})}} \text{ or } l(2)^{m+\frac{n}{12}}=L.$$

$$\therefore \downarrow,(l)=\downarrow,(L)-(m+\tfrac{n}{12})\downarrow,(2\cdot)$$

Ex. 14. Let the length $L=37\frac{1}{8}$ inches and sounds the note C, what lengths of the same string, with the same tension, will sound B and G sharp?

For the note B $m=0$ and $n=11$.

$$\therefore\ \downarrow,(l)=\downarrow,(37\tfrac{1}{8})-\tfrac{11}{12}\downarrow,(2\cdot)$$
$$=131170545,-63538492,+\downarrow,(10)$$
$$=67632053,\ +\downarrow,(10)$$
$$\downarrow,(1\cdot 96642704)=67632053,$$

$\therefore$ 19·6642704 is the length that will sound B.

For the note G sharp $m=1$ and $n=8$.

$$\therefore\ \downarrow,(l)=\downarrow,(37\tfrac{1}{8})-(1+\tfrac{8}{12})\downarrow,(\cdot 2)$$
$$=131170545,-115524530,+\downarrow,(10\cdot)$$
$$=15646015,+\downarrow,(10\cdot)$$
$$15646015,=\downarrow,(1\cdot 1696414)$$

$\therefore$ 11·696414 inches the length to sound G sharp.

Ex. 15. What is the solidity of a globe whose diamet(D) is 7896·2814 miles? *Ans.*

$$D^3\times\frac{\pi}{6}=\text{solidity}$$

$$\downarrow,(\text{Solidity})=3\downarrow,(D)+\downarrow,(\pi)-\downarrow,(6)$$
$$=3\downarrow,(D)+{}'64702958$$

4, $\qquad \downarrow,(\cdot 7896\cdot 2814)=\ '23682471$

3 $\qquad\qquad\qquad\qquad 3$

12, $\qquad\qquad\qquad '71047413$

'64702958 constant.

'135750371

$$\downarrow,(2^2)=138629436,$$

$$\downarrow,(1\cdot 02920905)=\ 2879065,$$

$$\frac{1029209050000\cdot}{4}=257302262500\cdot \text{ cubic miles.}$$

Ex. 16. What is the diameter of a globe the solid conte(of which is 8000· cubic feet?

$$\downarrow,(8000\cdot)=3\downarrow,(D)+{}'64702958$$
$$\therefore\ \downarrow,(D)=\frac{\downarrow,(8000)-{}'(64702958)}{3}$$

$$\downarrow,(8\cdot)\ =207944154,$$
$$\downarrow,(10^3)=690775527,$$

898719681,

64702958,

3)963422639,

321140880,

$$\begin{array}{r} 321140880, \\ \downarrow, (10) = \underline{'230258509} \\ \downarrow, (2{\cdot}48140158) = \quad 90882371, \end{array}$$

$\therefore \quad D = 24{\cdot}8$ nearly.

OTHERWISE, THUS,

$$\downarrow, (D) = \frac{\downarrow, (8000)}{3} - {}'21567653$$

$$= \frac{\downarrow, (8)}{3} + \frac{\downarrow, (10^3)}{3} - {}'21567653$$

$$= \frac{\downarrow, (8)}{3} + 21567653, + \downarrow, (10)$$

$$\frac{\downarrow, (8)}{3} = 69314718,$$

$$\underline{21567653,}$$

$$\downarrow, (2{\cdot}48140158) = 90882371,$$

(92) (93) (94).

Ex. 17. Find the area of an ellipse, whose transverse diameter (t) is 24·3 and conjugate (c) = 18·4 feet.

$$\text{Area} = \frac{t \times c \times \pi}{4};$$

$$\therefore \quad \downarrow, (\text{Area}) = \downarrow, (t) + \downarrow, (c) + \downarrow, (\pi) - \downarrow, (4)$$

$$\begin{array}{lll} 1, & \downarrow, (2{\cdot}43) & = \; 88789120, \\ \frac{1,}{2,} & \downarrow, (1{\cdot}84) & = \; 60976550, \\ & \downarrow, \left(\frac{\pi}{4}\right) & \quad \bar{1}75843553 \text{ ar. co.} \\ \hline & \downarrow, (3{\cdot}5116718) & = 125609223, \end{array}$$

$\therefore \quad \text{Area} = \; 351{\cdot}16718$

Ex. 18. Determine the solid content of a cylinder whose altitude (a) is 72·3 feet, and the diameter of its base 24·5 (b) feet. *Ans.* 34084·6642 feet.

$$\text{Solidity} = b^2 \times \frac{\pi}{4} \times a;$$

$$\therefore \quad \downarrow, (\text{Solidity}) = 2 \downarrow, (b) + \downarrow, \left(\frac{\pi}{4}\right) + \downarrow, (a)$$

1, $\quad 2\downarrow,(2.45) = 179217600,$

$\frac{2}{2,}$ $\quad \downarrow,\left(\frac{\pi}{4}\right) \quad \bar{1}75843553$ ar. co.

$\frac{2,}{4,}$ $\quad \downarrow,(\cdot 723) \quad \bar{1}67565391$ ar. co.

$$\downarrow,(3\cdot 408466 42) = 122626544,$$

$$\therefore \text{ Solidity} = 34084\cdot 6642.$$

Ex. 19. Find the solidity of a cone the diameter of whose base is 20·3 feet (d) and altitude 25·2 feet (a).

Ans. 2718·7 nearly.

$$\text{Solidity} = d^2 \times \frac{\pi}{4} \times \frac{a}{3} = ad^2 \times \frac{\pi}{12}$$

$$\therefore \downarrow,(\text{Solidity}) = \downarrow,(a) + 2\downarrow,(d) + \downarrow,\left(\frac{\pi}{12}\right)$$

$$= \downarrow,(a) + 2\downarrow,(d) + \text{'}134017676$$

1, $\quad 2\downarrow,(2\cdot 03) = 141607140,$

$\frac{2}{2,}$ $\quad \downarrow,(2\cdot 52) = 92425890,$

$\frac{1,}{3,}$ $\quad \downarrow,\left(\frac{\pi}{12}\right) \quad \bar{1}865982324$ ar. co.

$$\downarrow(2\cdot 71869919) = 100015354,$$

$$\therefore \quad \text{Solid content} = 2718\cdot 69919.$$

Ex. 20. The transverse or fixed axis of a prolate spheroid, $t = 185$, and the conjugate or revolving axis $c = 145$; what is the solidity? *Ans.* 2036602·64

$$\text{Solidity} = \frac{\pi}{6} c^2 t;$$

$$\therefore \downarrow,(\text{Solidity}) = 2\downarrow,(c) + \downarrow,(t) + \downarrow,\left(\frac{\pi}{6}\right)$$

2, $\quad 2\downarrow,(1\cdot 45) = 74312702,$

$\frac{2}{4,}$ $\quad \downarrow,(1\cdot 85) = 61518560,$

$\frac{2,}{6,}$ $\quad \downarrow,\left(\frac{\pi}{6}\right) \quad \bar{1}35297042$ ar. co.

$$\downarrow,(2\cdot 03660264) = 71128304,$$

$$\therefore \text{ Solidity} = 2036602\cdot 64$$

Ex. 21. Determine the content of an oblate spheroid whose fixed or minor axis $(c) = 145\cdot$, and whose revolving or major axis $(t) = (185)$.

$$\text{Solidity} = \frac{\pi}{6} \times t^2 c\,;$$

$$\therefore \downarrow, (\text{Solidity} = 2 \downarrow, (t) + \downarrow, (c) + \downarrow, \left(\frac{\pi}{6}\right)$$

$$\begin{array}{lrl}
2, & \downarrow, (1\cdot45) = & 37156351, \\
2, & 2 \downarrow, (1\cdot85) = & 123037120, \\
\frac{2}{4,} & \downarrow, \left(\frac{\pi}{6}\right) & \bar{1}35297042 \text{ ar. co.} \\
\hline
6, & \downarrow, (2\cdot5984238) = & 95490513, \\
 & \therefore \ \text{Solidity} = & 2598423\cdot8
\end{array}$$

Ex. 22. What is the value of

$$\sqrt{26961\cdot5586^2 + 4544\cdot21276^2}\,?$$

$26961\cdot5586 = 10^4 2 \downarrow 29867987$, square $= 10^8 2^2 \downarrow 59735974$,
$4544\cdot21276 = 10^3 2^2 \downarrow 12756017$, square $= 10^6 2^4 \downarrow 25512034$,

Then the given expression becomes the square root of

$$10^8 2^2 \downarrow 25512034, \left\{ \frac{\downarrow 59735974,}{\downarrow 25512034,} + \frac{10^6 2^4}{10^8 2^2} \right\} \text{ equal}$$

$$10^4 2 \downarrow 12756017, \left\{ \downarrow 34223940, + \frac{2^2}{10^2} \right\}^{\frac{1}{2}}$$

$$\text{but} \quad \downarrow 34223940, = 1\cdot40817734$$

$$\text{and} \quad \frac{2^2}{10^2} = \cdot04$$

$$\therefore \quad \downarrow, (1\cdot44817734) = 37030570,$$

$$\therefore \quad 10^4 2 \downarrow 12756017, \frac{\downarrow 37030570}{2} = 10^4 2 \downarrow 31271302$$

$$= 10^4 2 (1\cdot36712907)$$

$$= \quad 27342\cdot5814$$

108. This and the next example should receive particular attention.

Ex. 23. What is the value of

$$\sqrt{269\cdot615586^2 + 4544\cdot21276^2}\,?$$

$269\cdot615586 = 10^2 2 \downarrow 29867987$, square $= 10^4 2^2 \downarrow 59735974$,
$4544\cdot21276 = 10^3 2^2 \downarrow 12756017$, square $= 10^6 2^4 \downarrow 25512034$,

Then the given expression is reduced to the square root of

$$10^4 2^2 \downarrow 25512034, \times \left\{ \frac{\downarrow 59735974,}{\downarrow 25512034,} + \frac{10^6 2^4}{10^4 2^2} \right\}, \text{ equal}$$

$$10^2 2 \downarrow 12756017, \quad \left\{ \downarrow 34223940, + 10^2 2^2 \right\}^{\frac{1}{2}} \text{ equal}$$

$$10^3 2^2 \downarrow 12756017, \quad \left\{ \frac{1}{10^2 2^2} \downarrow 34223940, + 1 \right\}^{\frac{1}{2}};$$

$$\text{but} \quad 34223940, = (1\cdot40817734)$$

$$\text{and } \downarrow, \left(\frac{1\cdot40817734}{10^2 2^2} + 1 \right) = 351428,$$

$$\therefore \quad 10^3 2^2 \downarrow 12756017, \downarrow \frac{351428,}{2} = 10^3 2^2 \downarrow 12931731,$$

$$= 4552\cdot20464 \text{ the value required.}$$

With tables I. and II. the process requires little mental labour, each step is registered to prevent obscurity.

Ex. 24. Steam of 60 lbs. pressure to the square inch has a temperature of 295°·6 Fahrenheit, how many units of heat does it contain according to Regnault?

$$\begin{aligned} \text{Units of heat} &= 1091\cdot7 + (t^\circ - 32)\cdot305 \\ &= 1091\cdot7 + \cdot305(263\cdot6) \\ &= 1172\cdot098 \end{aligned}$$

Ex. 25. What is the pressure of steam in pounds to the square inch at the temperature of 295°·6 Fahrenheit?

Let P be the required pressure, then by the usual empirical formula

$$P = \left(\frac{T}{202} + \cdot52 \right)^6$$

T being the temperature of the steam in degrees of Fahrenheit's thermometer.

$$\frac{295\cdot6}{202} + \cdot52 = 1\cdot99326732$$

$$6 \downarrow, (1\cdot99326732) = 413865114, = \downarrow, (\cdot62707979) + \downarrow, (10^2)$$

$$\therefore \quad P = 62\cdot7 \text{ lbs. nearly.}$$

Ex. 26. How many cubic feet of steam of 55 lbs. pressure on the square inch will a cubic foot of water produce?

The volume V, may be found from the empirical formula.

$$\frac{V}{\downarrow^1 2,} - 10 = \frac{17000}{P^{\frac{40}{43}}}$$

P being the pressure in lbs.

$$\frac{40}{43}\downarrow,(P)=\frac{40}{43}(400733319,)$$
$$=372775181,\text{ take}$$
$$\downarrow,(170)=\underline{513579838},\text{ from}$$
$$\downarrow,(4\cdot0879618)=140804657,$$
$$\therefore\ \frac{17000}{P^{\frac{40}{43}}}=408\cdot79618$$
$$\therefore\ V=418\cdot79618\overset{1}{\downarrow}2,=506\cdot74338$$

So that a cubic foot will make 507· cubic feet of steam of 55 lbs. pressure on the square inch.

Ex. 27. From what depth will a steam-engine of 40 horse-power (*HP.*), raise 36 tons of coals per hour?

109. A unit of work is equal that amount of labour required to raise one pound, in the direction of the plumb-line, through the space of 1 foot. If 8 lbs. be raised 7 feet, in the direction of the plumb-line, 56 units of work will be performed. Should it take the same force to move a railroad carriage on a level rail and the carriage to be moved over 100 feet, then 800 units of work would be expended in this labour. Units of work thus considered do not involve time as an element; however we have a larger unit that involves time; namely 33000 lbs. raised a foot high in a minute or 550 lbs. raised a foot high in a second. This last unit is technically termed a horse-power.

∴ 40 horse-power $=33000\times40=132000$ units of work performed in an hour.

36 tons $=80640$ lbs. to be raised in an hour.

$$\frac{1320000}{80640}=57\cdot3\text{ feet, the required depth.}$$

110. A railway carriage only requires a pressure of $\frac{1}{280}$ part of its weight to put it in motion, or about 8 lbs. to the ton; the fraction $\frac{1}{280}$ is called the coefficient of friction. When a cart is drawn on a common road, the resistance to friction is between $\frac{1}{25}$ and $\frac{1}{32}$ of the whole load, $\frac{1}{25}$ and $\frac{1}{32}$ are termed coefficients of friction in this case. If a horse draws a ton on a common road, when pulling with the force of 80 lbs., then $\frac{1}{28}$ is the coefficient of friction.

Ex. 29. What must be the effective horse-power of a locomotive engine which moves at the uniform velocity of

25·5 miles an hour upon a level rail, the weight of the train being 52·3 tons, and the friction 7·7 lbs. a ton, the resistance of the atmosphere being neglected?

$$52{\cdot}3 \times 7{\cdot}7 = \text{resistance to friction};$$

$$\frac{25{\cdot}5 \times 5280}{60} = \text{feet passed over in a minute};$$

$$\frac{25{\cdot}5 \times 5280}{60} \times 52{\cdot}3 \times 7{\cdot}7 = \text{units of work due to friction}$$
$$= 25{\cdot}5 \times 880 \times 52{\cdot}3 \times 7{\cdot}7.$$

Since the velocity of the train is uniform, the work of the resistance must be equal to the effective work of the engine.

$$\frac{25{\cdot}5 \times 880 \times 52{\cdot}3 \times 7{\cdot}7}{33000} = \frac{25{\cdot}5 \times 8 \times 14{\cdot}1 \times 7{\cdot}7}{100} =$$
$$27{\cdot}68535 \text{ horse-power.}$$

Ex. 30. In a condensing engine the length of stroke (L), $= 6{\cdot}4$ feet, the steam cut off at $l = 2{\cdot}8$ feet, and the pressure of the steam in the cylinder (P) $= 48$ lbs. on the square inch; how many units of work is due to 1 square inch of this piston in a stroke?

$$\text{Units of work} = 4{\cdot}8 \times 2{\cdot}8 \left(1 + \frac{\downarrow,(L) - \downarrow,(l)}{10^8}\right);$$

$$\begin{array}{lll} \text{but,} & \downarrow,(6{\cdot}4) = & 185629790, \\ \text{and} & \downarrow,(2{\cdot}8) = & \underline{102604150}, \\ & & 83025640, \end{array}$$

$$\therefore\ 4{\cdot}8 \times 2{\cdot}8 \times 1{\cdot}83 = 245{\cdot}952 \text{ units.}$$

This last continued multiplication may be performed by dual logarithms. The effectual force on a square inch throughout the stroke $= \frac{4{\cdot}8 \times 2{\cdot}8}{6{\cdot}4}(1{\cdot}8302564) =$

$$2{\cdot}1(1{\cdot}8302564) = 38{\cdot}4353844 \text{ lbs.}$$

Ex. 31. The pressure of steam upon the piston is 65 lbs. to the square inch, the length of the stroke $= 11{\cdot}6$ feet, the steam is cut off when 2·4 feet of the stroke is made; find the units of work done on each square inch of the piston.

Logarithm of the required units of work $=$

$$\downarrow,(65) + \downarrow,(2{\cdot}4) + \downarrow,\left(1 + \frac{\downarrow,(11{\cdot}6) - \downarrow,(2{\cdot}4)}{0^8}\right)$$

$$\frac{\downarrow,(11{\cdot}6) - \downarrow,(2{\cdot}4)}{10^8} = 1{\cdot}57553639 \text{ to which add 1.}$$

Then,

$$\begin{array}{lrl} \downarrow, (2{\cdot}57553639) = & 94605786, & \\ \downarrow, (2{\cdot}4) \qquad = & \_87546870, & \\ \downarrow, ({\cdot}65) & \overline{1}56921701 & \text{ar. co.} \\ \hline \downarrow, (4{\cdot}01783636) = & 139074357, & \end{array}$$

$\therefore$ 401·783636 units of work required.

See "Dual Arithmetic, a new Art," page 147.

Ex. 32. Suppose the cylinder in the last example to be 88 inches in diameter, and the piston to make 16 double strokes a minute; that is, 16 revolutions of the crank; what is the horse-power of the engine?

$$\text{Horse power} = \frac{(88)^2 \times \frac{\pi}{4} \times 32 \times 401{\cdot}178}{33000}, \text{ then}$$

$$\begin{array}{lll} 2, & \downarrow, ({\cdot}88) = & '12783339 \\ \underline{2} & & \underline{\qquad\qquad 2} \\ 4, & & '25566678 \\ & \downarrow, \left(\frac{\pi}{4}\right) = & '24156447 \\ 2, & \downarrow, ({\cdot}32) = & \_'113943429 \\ 2, & \downarrow, (4{\cdot}01783636) = & \overline{1}860925643 \text{ ar. co.} \\ \hline & & 324592197 \\ 5, & \downarrow, ({\cdot}33) & 1{\cdot}10866269, \text{ divisor.} \\ \hline & \downarrow (2{\cdot}36964634) = & 86274072, \end{array}$$

$\therefore$ 2369·6 the required H.P.

111. When the motion of a body is uniform, it is evident that the space described is equal to the units of time multiplied by the units of space passed over in each unit of time, or

$$\text{space} = \text{time} \times \text{velocity}.$$

The resistance of the atmosphere being neglected, when bodies fall freely near the surface of the earth, the force of gravity gives them equal increments of velocity in equal intervals of time, since the force of the attraction of the earth is considered constant. It is also stated that experiments show that at the end of one second the velocity of a falling body is nearly $32\frac{1}{6}$ feet a second.

$$\downarrow, (32\tfrac{1}{6}) = 347093071,$$

Taking these premises for granted, the velocity of a falling body at the end of two seconds $= 2 \times 32\frac{1}{6}$ feet, at the end of 3 seconds $= 3 \times 32\frac{1}{6}$ feet; or generally the velocity v, = time t, $\times 32\frac{1}{6}$; or

$$v = t \times 32\frac{1}{6}$$

$$\therefore \downarrow, (v) = (t) + 347093071, \qquad \text{(I.)}$$

A similar process of reasoning will show that the velocity at the end of $\frac{1}{2}$ a second will be $16\frac{1}{12}$ feet, and this being the mean velocity during a second, a body must describe $16\frac{1}{12}$ feet in the first second. The velocity of a body at the end of 2 seconds being $32\frac{1}{6} \times 2$, the mean velocity for the time, in this case, being $32\frac{1}{6}$, hence the body must describe $4 \times 16\frac{1}{12}$ feet in 2 seconds; again, the velocity of a body at the end of 3 seconds $= 32\frac{1}{6} \times 3$ and the mean velocity for this time, or the velocity for $1\frac{1}{2}$ second $= 32\frac{1}{6} \times \frac{3}{2}$.

$$\therefore 32\tfrac{1}{6} \times \tfrac{3}{2} \times 3 = 16\tfrac{1}{12} \times 3^2 \text{ the space}$$

passed over in three seconds. Generally the space s, passed over in the time t, $= 16\frac{1}{12} \times t^2$ for the mean or uniform velocity acquired in half the time $= 32\frac{1}{6} \times \frac{t}{2}$

$$\text{and } 32\tfrac{1}{6} \times \frac{t}{2} \times t = 16\tfrac{1}{12} \times t^2;$$

$$\therefore s = 16\tfrac{1}{12} \times t^2$$

$$\text{or } \downarrow, (s) = 2 \downarrow, (t) + 277778353, \qquad \text{(II.)}$$

Ex. 33. Through what space will a body fall in 2·47 seconds?

$$\begin{aligned} \downarrow, (2{\cdot}47) \quad &= 90421810, \\ &\quad\;\; 2 \\ &\overline{180843620,} \\ &\underline{277778353,} \text{ constant} \\ \downarrow, (98{\cdot}1227924) &= 458621973, \end{aligned}$$

Also, $\downarrow, (s) = 2 \downarrow, (t) + 47519844, + \downarrow, (10)$. (II.)

Ex. 34. From what height would a body fall to acquire a velocity of 217 feet? *Ans.* 1989·67521 feet.

From (I.) $\downarrow, (t) = \downarrow, (v) - 347093071$, and
from (II.) $\downarrow, (t) = \frac{1}{2} \downarrow, (s) - 138889177$,

$$\therefore \tfrac{1}{2} \downarrow, (s) = \downarrow, (v) - 347093071, + 138889177,$$

$$\therefore \downarrow, (s) = 2 \downarrow, (v) - 416407788,$$

$$= 2 \downarrow, (v) - \downarrow, (10^2) + 44109230, \text{ (III.)}$$

Then the solution of the example may be arranged thus:—

$$\begin{array}{lll}
2, & \downarrow, (2{\cdot}17) & = 77472710, \\
\underline{2} & & \quad\underline{\qquad\quad 2} \\
4, & & 154945420, \\
\underline{2} & & \quad \underline{\;44109230,} \\
2, & \downarrow, (19{\cdot}8967521) & = 299053650,
\end{array}$$

Putting g for $32\frac{1}{6}$, (III.) may be also established as follows,

$$\text{Since } v=gt \text{ and } s=t^2\times\frac{g}{2}, \text{ therefore,}$$

$$t=\frac{v}{g} \text{ and } s=\frac{v^2}{g^2}\times\frac{g}{2}=\frac{v^2}{2g}.$$

That is the space passed over is equal to the square of the velocity divided by $32\frac{1}{6}\times 2$. The resistance of the atmosphere is not taken into account, nor do we say that the number $32\frac{1}{6}$ is correct.

Ex. 35. In what time will a body fall $2\frac{1}{2}$ miles?

Ans. 90·593841 seconds.

$$2{\cdot}5 \text{ miles}=5280\times 2\tfrac{1}{2}=132000 \text{ feet.}$$

$$\text{From (II.)} \quad \downarrow, (t)=\frac{\downarrow, (s) + {}'277778353}{2}$$

$$\begin{array}{lll}
 & \downarrow, (1{\cdot}32) & = 27763170, \\
2)\underline{5}, \text{ and 1 over} & & \underline{230258509,} \\
2, & & 258021679, \\
 & & \underline{{}'277778353} \text{ constant} \\
 & & 2)\;\underline{{}'19756674} \\
 & \downarrow ({\cdot}90593841) & = {}'9878337
\end{array}$$

$\therefore$ the time=90·593841 seconds.

112. To determine the units of work accumulated in a body moving with a given velocity it is only necessary to find the height from which a body must fall to acquire the given velocity, then the required units of work=the height found in feet $\times$ the weight of the body in lbs.

Ex. 36. How many units of work are accumulated in a round shot weighing 218 lbs. moving with the velocity of 1000 feet a second? *Ans.* 3388602.

$$218\times\frac{(1000)^2}{64\frac{1}{3}}=\frac{654000000}{193}=3388602.$$

If w=the weight, v=the velocity, and U=the accumulated units of work in the moving body,

Then, $\downarrow,(U)=2\downarrow,(v)+\downarrow,(w)+44109229,-\downarrow,(10^2)$

Ex. 37. How many units of work are accumulated in a cast-iron shot 12 inches diameter weighing 235·851319 lbs. when moving with a velocity of 1234·321 feet a second?

Ans. 5585454·12 units.

3,		
2		
6,	$2\downarrow,(1{\cdot}234321)$	=42104204,
2,		
8,	$\downarrow,(2{\cdot}35851319)$	=85803141,
2,		
6,		44109229,
	$\downarrow,(558545412)$	=172016574,

Ex. 38. Suppose the length of the bore of a gun to be 16 times the diameter of the ball, what is the mean pressure on a square inch of the great circular section of the ball perpendicular to the axis of the gun carrying the shot of the last example? *Ans.* 3086·6409 lbs.

Let x be the mean pressure on each square inch of cross section, then the units of work developed will be

$$x\times 12^2\times\frac{\pi}{4}\times 16, \text{ which is therefore}=5585454{\cdot}12.$$

$$\therefore\ x=\frac{5588454{\cdot}12}{144\times 16\times\frac{\pi}{4}}.$$

6,	$\downarrow,(5{\cdot}58545412)$	=$\bar{1}$72016574,
'2	$\downarrow,(1{\cdot}44)$	$\bar{1}$63535690 ar. co.
'1	$\downarrow,(1{\cdot}6)$	$\bar{1}$52999637 ar. co.
3,	$\downarrow,\left(\frac{\pi}{4}\right)$	24156447, comma moved.
	(3·0866409)	=112708348,

Ex. 39. Taking the data of the previous examples, what length of bore would be occupied by 1260 ounces of gunpowder, which is about $\frac{1}{3}$ the weight of the ball?

Ans. 1·71214717 feet.

Since a cubic foot of gunpowder weighs 937 ounces, $\frac{1260}{937}$=the content of the powder in cubic feet.

Let x be the required length of bore, then $1^2 \times \frac{\pi}{4} \times x =$ the content in cubic feet also;

$$\therefore \; x = \frac{1260\cdot}{\frac{\pi}{4} \times 937}.$$

3,	$\downarrow, (1\cdot 26)$	$= 23111170$,	
—	$\downarrow, \left(\frac{\pi}{4}\right)$	24156447,	comma changed.
3,	$\downarrow, (\cdot 937)$	6507209,	comma changed.

$$(1\cdot 71214717) = 53774826,$$

Ex. 40. With a charge of gunpowder filling 15 inches of the bore of a gun 16 feet long and 12 inches diameter, a cast-iron round shot weighing 235 lbs. is projected with an initial velocity of 1000 feet a second; what is the pressure on a square inch before expansion, supposing the force developed by exploding the powder to be confined to the space it occupies, and the expansion to take place according to Boyle's law? *Ans.* 7285·87 lbs.

The area of a circle 12 inches diameter $= 113$ square inches nearly.

$\frac{(1000)^2}{64\frac{1}{3}} \times 235\cdot =$ the units of work accumulated in the shot through the initial force and expansion of the powder.

$\frac{(1000)^2 \times 235}{64\frac{1}{3} \times 113} = 32326$, the units of work to each square inch.

Let z be the required pressure, then by examples 30 and 31,

$$\downarrow, (32326\cdot) = \downarrow, (z) + \downarrow, (1\tfrac{1}{4}) + \downarrow, \left(1 + \frac{\downarrow, (16) - \downarrow, (1\tfrac{1}{4})}{10^8}\right)$$

$$\text{or} \quad 1\cdot 25z\left(1 + \frac{\downarrow, (16) - \downarrow, (1\cdot 25)}{10^8}\right) = 32326\cdot$$

$$\text{or} \quad 1\cdot 25z(3\cdot 549944519) = 32326\cdot$$

$$\therefore \; z = 7285\cdot 87.$$

Hence the gun will not burst, since the crushing strength of cast-iron is about 28750 lbs. to the square inch.

113. The subjoined dual logarithms to 18 places of figures may be found useful.

$$\downarrow,\ (2)=\ \ 6\,9\,3\,1\,4\,7\,1\,8\,0\,5\,5\,9\,9\,4\,5\,3\,3,$$
$$\downarrow,\ (3)=1\,0\,9\,8\,6\,1\,2\,2\,8\,8\,6\,6\,8\,1\,0\,9\,6\,9,$$
$$\downarrow,\ (4)=1\,3\,8\,6\,2\,9\,4\,3\,6\,1\,1\,1\,9\,8\,9\,0\,6\,6,$$
$$\downarrow,\ (5)=1\,6\,0\,9\,4\,3\,7\,9\,1\,2\,4\,3\,4\,1\,0\,0\,3\,5,$$
$$\downarrow,\ (6)=1\,7\,9\,1\,7\,5\,9\,4\,6\,9\,2\,2\,8\,0\,5\,5\,0\,2,$$
$$\downarrow,\ (7)=1\,9\,4\,5\,9\,1\,0\,1\,4\,9\,0\,5\,5\,3\,1\,3\,2\,9,$$
$$\downarrow,\ (8)=2\,0\,7\,9\,4\,4\,1\,5\,4\,1\,6\,7\,9\,8\,3\,5\,9\,9,$$
$$\downarrow,\ (9)=2\,1\,9\,7\,2\,2\,4\,5\,7\,7\,3\,3\,6\,2\,1\,9\,3\,8,$$
$$\downarrow,\ (10)=2\,3\,0\,2\,5\,8\,5\,0\,9\,2\,9\,9\,4\,0\,4\,5\,6\,8,$$

Ex. 41. Find by a direct process the common logarithm of 7; or solve the equation,

Given $10^x = 7\cdot$; find x.

Such equations could not be solved, before dual arithmetic was invented, without much labour and a great amount of dodging and guessing.

$$x \downarrow, (10) = \downarrow, (7)$$
$$\therefore\ x = \frac{\downarrow, (7)}{\downarrow, (10)} = \frac{194591014,}{230258509,} =$$

$\cdot 84509804$ the common logarithm of $7\cdot$

$$7 = {}'3'4'0'3'9'2'0'3\,' \uparrow (10)$$
$$\therefore\ \downarrow, (7) = {}'3'4'0'3'9'2'0'3\,' \uparrow + \downarrow, (10)$$
$$= {}'35667495 + 230258509, = 194591014,$$

See "Dual Arithmetic, a new Art," pp. 44, 47, 207.

Ex. 42. The number $1871\cdot$ is a prime number, find its common logarithm by a direct calculation, or, in other terms, solve the equation

$10^x = 1871$. To find x.

$$x \downarrow, (10) = \downarrow, (1871)$$
$$\therefore\ x = \frac{\downarrow, (1871)}{\downarrow, (10)} = \frac{753422813,}{230258509,} = 3\cdot 27207378$$

Then $3\cdot 27207378$ is the common logarithm of 1871.

$$1871$$
$$\|$$
$${}'1'0'1'1'1'4'4'4\,1 \overset{\uparrow}{\underset{\downarrow,}{}}3\,0,4,$$
$$\|$$
$${}'6667414 + \downarrow, (2) + \downarrow, (10^3)$$

Ex. 43. What natural number answers to the common logarithm 3·27207378, or, in other terms, solve the equation

$$10^{3\cdot27207378}=y, \text{ by a direct calculation.}$$

$$(3\ 27207378)\downarrow,(10)=\downarrow,(y)$$

$$\therefore\ 753422813,=\downarrow,(y)$$

$$\downarrow,(y)=3\downarrow,(10)+\downarrow,(2)+{}'6667414;$$

$$\text{but } {}'6667414={}'0'6'6'3'6'9'1'2\uparrow=\cdot9355$$

$$\therefore\ y=\cdot9355\times10^3\times2=1871;$$

the required natural number.

114. $\epsilon=2\cdot71828182$, the base of the hyperbolic system of logarithms.

Ex. 44. Find the hyperbolic logarithm of 10· or solve the equation, given

$$\epsilon^x=10. \quad \text{To find } x, \text{ by a direct calculation}$$

$$x\downarrow,(\epsilon)=\downarrow,(10)$$

$$\therefore\ x=\frac{\downarrow,(10)}{\downarrow,(\epsilon)}=\frac{230258509,}{100000000,}=2\cdot30258509$$

$$\epsilon=2\cdot718281828=2\downarrow3,2,1,0,2,2,1,2,$$

$$\therefore\ \downarrow,(\epsilon)=10000000,$$

Ex. 45. Required the natural number corresponding to the hyperbolic logarithm 1·14472989; or, in other terms, solve the equation

$$(2\cdot718281828)^{1\cdot14472989}=y;$$

that is, find y by a direct calculation.

$$1\cdot14472989\downarrow,(2\cdot718281828)=\downarrow,(y).$$

$$\therefore\ 114472989,=\downarrow,(y)=\downarrow,(2)+45158271,$$

$\therefore\ y=2\downarrow4,7,0,6,8,9,6,8,=3\cdot1415927=\pi$ the required natural number. *See* Note B.

[illegible] be *n* decimals in [illegible] placed in the *n*th p[illegible]

[illegible]

[illegible] and also in the sec[illegible] [illegible] is employed; [illegible] occupied by cipher[illegible]

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[illegible]

[illegible] introduced, unl[illegible] [illegible] apart from [illegible] [illegible] in the gene[illegible] [illegible] branch.

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S 8

A.

new art, and its developments
res and differing essentially in
ts, the binomial theorem, the
Taylor, Lagrange, Laplace, &c.,
to those expert in use of these
of this art rests on these sum-
The art and science of Dual
independently established. The
s is not subject to the same
quantities. A great variety of
ressing a known or unknown
ing the magnitude to vary or
small magnitude which together
a magnitude expressed under the

let $u=a\left(1+\frac{1}{x}\right)$, then

However, to develop $x\left(1-\frac{1}{x}\right)$

whether x be great or small.
put $u=\log x$, then

Maclaurin's theorem fails.
thm of x to any base is easily

theorem may be written

$$\ldots \frac{d^3 u}{dx^3}\frac{h^3}{1.2.3}+\ldots$$

the basis of his theory of analy-
may be given to x, that will
ible. For example, let

$(x+h)$

$$\ldots\frac{h^2}{2x^2}+\frac{h^3}{3x^3}-\frac{h^4}{4x^4}+\ldots$$

$$=-\infty+\infty-\infty+\ldots$$

ping $\log h$.

NOTES

NOTE A.

In analysing the elements of this new art, and its developments, which are without exceptions or failures and differing essentially in their nature from all other developments, the binomial theorem, the theorems of Stirling (or Maclaurin), Taylor, Lagrange, Laplace, &c., have been introduced to accommodate those expert in use of these theorems; yet no principle or portion of this art rests on these cumbersome and deficient performances. The art and science of Dual Arithmetic are founded on principles independently established. The continued product of derived quantities is not subject to the same irregularities as the sum of derived quantities. A great variety of dual developments may be found expressing a known or unknown number or magnitude without supposing the magnitude to vary or supposing it made up of a large and a small magnitude added together.

By the theorem called Maclaurin's, a magnitude expressed under the form $a\left(1+\frac{1}{x}\right)$ cannot be developed; for let $u=a\left(1+\frac{1}{x}\right)$, then

$$\frac{du}{dx}=-\frac{a}{x^2},$$

which becomes infinite when $x=0$. However, to develop $a\left(1+\frac{1}{x}\right)$ by dual arithmetic is an easy matter, whether x be great or small. Again, to develop $\log x$, by this theorem, put $u=\log x$, then

$$\frac{du}{dx}=\frac{1}{x},$$

which becomes infinite when $x=0$, and Maclaurin's theorem fails.

But by dual arithmetic the logarithm of x to any base is easily developed and determined.

If $u=f(x)$ and $u'=f(x+h)$, Taylor's theorem may be written

$$u'=u+\frac{d.u}{dx}\frac{h}{1}+\frac{d^2.u}{dx^2}\frac{h^2}{1.2}+\frac{d^3.u}{dx^3}\frac{h^3}{1.2.3}+\ldots$$

a theorem which Lagrange has made the basis of his theory of analytic functions, although particular values may be given to x, that will render this form of development impossible. For example, let

$$f(x+h)=\log(x+h)$$

$$\text{then } \log(x+h)=\log x+\frac{h}{x}-\frac{h^2}{2x^2}+\frac{h^3}{3x^3}-\frac{h^4}{4x^4}+\ldots.$$

when $x=0$,

$$\log(0+h)=\log h=-\infty+\infty-\infty+\ldots.$$

a result of no use whatever in developing $\log h$.

Again, let $f(x)=x^2+x^{\frac{8}{3}}+x^3\log x=u$, then

$$f(x+h)=(x+h)^2+(x+h)^{\frac{8}{3}}+(x+h)^3\log(x+h);$$

$$\frac{d.u}{dx}=2x+\frac{8}{3}x^{\frac{5}{3}}+3x^2\log x+x^2=0+0+0\times-\infty+0; \text{ when } x=0;$$

$$\frac{d^2.u}{dx^2}=2+\frac{40}{9}x^{\frac{2}{3}}+6x\log x+3x+2x=2+0+0\times-\infty+0;$$

$$\frac{d^3.u}{dx^2}=\ +\frac{80}{27}x^{-\frac{1}{3}}+6\log x+6+5=\infty-\infty+11;$$

results which are not very intelligible, and of very little use in finding the value of x in the equation

$$x^2+x^{\frac{8}{3}}+x^3\log x=213{\cdot}8570854=a,$$

any given number. The value of x is readily found by the dual method of solving equations, while no other known development will apply.

$$x='4'4'3'6\ {}_4\!\uparrow \tfrac{9}{2} \downarrow^3 2,; \qquad x=\downarrow 15,7,6,7,3,0,5,2,;$$

$$x=\tfrac{9}{2}\downarrow 0,0,1,9,5,5,1,4,; \quad x=3\downarrow 4,2,6,2,8,1,3,6,$$

$$x=4\downarrow 1,2,4,5,3,7,6,8,=4{\cdot}50880428{\cdot}$$

$$x=4{\cdot}50880428=\left\{\begin{array}{c} 4\downarrow 1,2,4,5,3,7,6,8, \\ 3\downarrow 4,2,6,2,8,1,3,6, \\ \tfrac{9}{2}\downarrow 0,0,1,9,5,5,1,4, \\ \&c. \end{array}\right\}=\downarrow^8 150603205,$$

Hyperbolic log $x=1{\cdot}5063205$; $x^{\frac{8}{3}}=(10)\ (2^2)\downarrow 3,4,1,4,7,4,6,5,$; $x^3=(10)\,(2^3)\downarrow 1,4,0,9,5,8,0,1,$ Without finding the value of x in natural numbers, these functions of x may be determined.

Let $u=\Psi(z)$ be a function to be developed when $z=F\{y+xf(z)\}$; x and y are supposed to be independent variables; z is evidently a function of x and y.

The general expression, $\dfrac{d^n u}{dx^n}=\dfrac{d^{n-1}\,(f(z))^n\dfrac{du}{dy}}{dy^{n-1}}$, being found by the process termed differentiation, and according to Stirling's theorem x is put $=0$ in the original function u, and in the derived functions

$$\frac{du}{dx},\ \frac{d^2u}{dx^2},\ \ldots\ldots\ \frac{d^nu}{dx^n}. \quad \text{Since } z=F\{y+xf(z)\},$$

on the condition $x=0$, $z=F(y)$ and as $u=\Psi(z)$

$$\therefore\quad \{u\}_{x=0}=\Psi(F(y)), \text{ which}$$

known function of y may be expressed by the characteristic ϕ:

then $\{u\}_{x=0}$ for the sake of brevity may be written $\phi(y)$

$$\{z\}_{x=0} \text{ becomes } F(y), \quad \therefore\quad f\{z\}_{x=0}=f(F(y))$$

may be expressed by $\Psi(y)$, and consequently

$$\left\{\frac{d^n.u}{dx^n}\right\}_{x=0}=\frac{d^{n-1}.(\Psi(y))^n\,\dfrac{d\phi(y)}{dy}}{dy^{n-1}}$$

then, according to Stirling's theorem,

$$u=\phi(y)+\Psi(y)\frac{d\phi(y)}{dy}\frac{x}{1}+\frac{d(\Psi(y))^2\frac{d\phi(y)}{dy}}{dy}\frac{x^2}{1.2}+\frac{d^2.(\Psi(y))^3\frac{d\phi(y)}{dy}}{dy^2}\frac{x^3}{1.2.3}+\ldots$$

which is the theorem of Laplace.

If $u=\Psi(z)$ and $z=y+xf(z)$, then according to Stirling's theorem, and the notation just established

$$\{z\}_{x=0}=F(y)=y, \text{ and } \{u\}_{x=0}=\Psi(F(y))=\Psi(y)=\phi(y);$$

$f\{z\}=f(F(y))=f(y)=\Psi(y)$, and Laplace's theorem becomes

$$u=\Psi(y)+f(y)\frac{d.\Psi(y)}{dy}\frac{x}{1}+\frac{d.(f(y))^2\frac{d.\Psi(y)}{dy}}{dy}\frac{x^2}{1\cdot 2}+\frac{d^2.(f(y))^3\frac{d\Psi(y)}{dy}}{dy^2}\frac{x^3}{1.2.3}+.$$

which is the theorem of Lagrange.

These theorems may be represented under many shapes, and receive numerous modifications. For example, if $x=1$, and p be put for $f(y)$; and q for $\frac{d.\Psi(y)}{dy}$, then Lagrange's theorem becomes

$$u=\Psi(y)+pq\frac{1}{1}+\frac{d.(p^2q)}{dy}\frac{1}{1.2}+\frac{d^2(p^3q)}{dy^2}\frac{1}{1.2.3}+\&c.$$

It was in this shape Lagrange delivered his theorem. Again, since $f(y)$ represents any function of y, it also may represent $y^{\circ}=1$, then $f(y)$, $(f(y))^2$, &c. $=1$. And as $f(y)$ becomes, in this case y°, $f(z)$ becomes $z^{\circ}=1$ also; then $z=y+xf(z)=y+x$, or $=y+h$ by putting h for x,

$$\therefore\ u=\Psi(z)=\Psi(y+h)=\Psi(y)+\frac{d^1\Psi(y)}{dy}\frac{h}{1}+\frac{d^2\Psi(y)}{dy^2}\frac{h^2}{1.2}+\ldots.$$

which is Taylor's theorem, from which Stirling's theorem may be derived, and from Stirling's theorem the Binomial Theorem may be established. But it ought to be observed that while Stirling's theorem establishes the Binomial theorem in the most general manner, it often fails to show that particular developments by the binomial theorem are true. Laplace has extended his theorem to functions of several variables, but this extension is of but very little real practical value; for the preceding theorems often become inapplicable from the complication of the processes that determine the successive steps; and often become inadequate, or fail, from the functions which are to be developed becoming infinite or indeterminate.

How cumbersome and uncertain the unwieldy developments of the theorems of Lagrange and Laplace are, compared with dual developments is best shown by example. To illustrate this statement, let it be required to find a number whose common logarithm is composed of the same consecutive digits.

The equation to be solved is $10^{\frac{z+1}{10}}=1+z$, find z.

$z=(-1)+10^{\frac{1}{10}}10^{\frac{z}{10}}$; $y=(-1)$; $z=10^{\frac{1}{10}}$; for by Lagrange's theorem

$$z=y+xf(z).$$

By putting F instead of Ψ, $u=F(z)=F(y+xf(z))=$

$$F(y)+\frac{d.F(y)}{dy}f(y)\frac{x}{1}+\frac{d.\left\{\frac{d.F(y)}{dy}(f(y))^2\right\}}{dy}\frac{x^2}{1\cdot 2}+\frac{d^2.\left\{\frac{d.F(y)}{dy}(f(y))^3\right\}}{dy^2}\frac{x^3}{1.2.3}+$$

We have to determine the simplest possible function of z, namely, z itself. $\therefore\ u=F(z)=z$, hence the nature of the function expressed by F becomes known; in this example $\therefore\ F(y)=y$, and $f(z)=10^{\frac{z}{10}}$, hence, also, the nature of the function expressed by f becomes known, $\therefore f(y)=10^{\frac{y}{10}}$. $x=10^{\frac{1}{10}}$.

$$F(y)=y=-1; \qquad (A).$$

$$\frac{d.F(y)}{dy}f(y)\frac{x}{1}=\frac{dy}{dy}f(y)\frac{x}{1}=\frac{dy}{{}_1dy}10^{\frac{y}{10}}10^{\frac{1}{10}}=10^{\frac{-1}{10}}10^{\frac{1}{10}}=+1; \qquad (B).$$

$$\frac{d.\left\{\frac{d.F(y)}{dy}(f(y))^2\right\}}{dy}\frac{x^2}{1.2}=\frac{d.\{(f(y))^2\}}{dy}\frac{x^2}{1.2}=\frac{d.10^{\frac{2y}{10}}}{dy}\frac{x^2}{1.2}=$$

$$\log 10\cdot 10^{\frac{2y}{10}}\cdot\frac{2}{10}\frac{x^2}{1.2}=\frac{2}{10}(\log 10)\,10^{\frac{2y}{10}}\frac{10^{\frac{2}{10}}}{1.2}=\frac{2}{10}(\log 10)\frac{1}{1.2},$$

when the function of y is substituted; (C).

$$\frac{d^2.\left\{\frac{d.F(y)}{dy}(f(y))^3\right\}}{dy^2}\frac{x^3}{1.2.3}=\frac{d^2.(f(y))^3}{dy^2}\frac{x^3}{1.2.3}=\left(\frac{3}{10}\right)^2\log(10)^2\frac{1}{1.2.3}; (D).$$

$$\frac{d^3.\left\{\frac{d.F(y)}{dy}(f(y))^4\right\}}{dy^3}\frac{x^4}{13.4}=\left(\frac{4}{10}\right)^3(\log 10)^3\frac{1}{1.2.3.4}; \qquad (E).$$

$$\frac{d^4.\left\{\frac{d.F(y)}{dy}(f(y))^5\right\}}{}\frac{x^5}{1.2.3.4.5}=\left(\frac{5}{10}\right)^4(\log 10)^4\frac{1}{1.2.3.4.5}; \qquad (F).$$

$$\therefore\ u=F(z)=A+B+C+D+E+F+\ldots\ldots$$

$$\therefore\ F(z)=z=\left(\frac{2}{10}\right)^1(\log)^1\frac{1}{1.2}+\left(\frac{3}{10}\right)^2(\log 10)^2\frac{1}{1.2.3}+$$

$$\left(\frac{4}{10}\right)^3(\log 10)^3\frac{1}{1.2.3.4}+\ldots$$

then by a laborious calculation z may be found $=\cdot 3712885742$

Hence $\log 1\cdot 3712885742=\cdot 13712885742$.

he value of z in the equation $10^{\frac{1+z}{10}}=1+z$, is readily obtained in a dual form; for $\frac{1+z}{10}\log 10=\log(1+z)$,

$$\text{or } \frac{\log 10}{10}=\frac{\log(1+z)}{1+z}.$$

Dispensing with the use of logarithms, therefore, by dual logarithms

$$\frac{\downarrow,(10)}{10}=\frac{\downarrow,(1+z),}{1+z}.$$

$$\downarrow,(10)=230258509,\text{ and }\frac{\downarrow,(10)}{10}=23025851.$$

$$\therefore\ (23025851.)(1+z)=\downarrow,(1+z),$$

It is easily observed that $\downarrow(1+z)$, is greater than $\downarrow 3$, and less than $\downarrow 4$, and in the following direct manner the complete development is obtained.

```
2|3|0|2|5|8|5|1
 |6|9|0|7|7|5|5
   |6|9|0|7|7|6
     |2|3|0|2|6
----------------
3 0|6 4|7 4|0 8 =↓ 3. . . . . . . . . =28593054
   |9 1|9 4|2 2
       |9 1|9 4
           |3 1
----------------
3 1 5 7 6|0 5 5 =↓ 0,3, . . . . . . . =  2985099
         |9 4 7                         --------
                                        31578153
----------------
3 1 5 7 5 1 0|8 =’0’0’0’0’3 ↑ . . . =      3000
            2|2                         --------
                                        31575153
----------------
3 1 5 7 5 0 8 6|=’0’0’0’0’0’0’7 ↑ . =        70
              1|                        --------
                                        31575083
----------------
3 1 5 7 5 0 8 7 =↓ 0,0,0,0,0,0,0,3, =         3
                                        --------
                                        31575086
```

$$\therefore\ z+1 = {}'3'0'7 \underset{5}{\updownarrow} 3,3, \overset{8}{\downarrow} 3, = 1\cdot37128857$$

$$\text{And } \therefore \log 1\cdot37128857 = \cdot137128857$$

It may be added that a dual development, and no other known development, has the capability to establish the coincidences

$$\begin{aligned}
\log 1\cdot3712885742385 42 &= \cdot1371288574238542\\
\log 237\cdot5812087593221 &= 2\cdot3758120875932 21\\
\log 3550\cdot260181586591 &= 3\cdot550260181586591\\
\log 46692\cdot46832877758 &= 4\cdot66924683287775 8\\
\log 576045\cdot6934135527 &= 5\cdot760456934135527\\
\log 6834720\cdot776754357 &= 6\cdot834720776754357\\
\log 78974890\cdot31398144 &= 7\cdot897489031398144\\
\log 895191599\cdot8267852 &= 8\cdot951915998267852
\end{aligned}$$

To compare a dual development with a development by Laplace's Theorem,

Let $v=\varepsilon^{m+n\cos v}$, find v^3 in terms of m and n. The given equation must be compared with $F\{y+xf(z)\}$, and $u=\Psi(z)$ with v^3.

$$v=z;\ m=y;\ n=x;$$

$$\therefore\ f(z)=f(v)=\cos v; \qquad \therefore\ f(F(y))=\cos F(y)=\cos F(m).$$

$$u=\Psi(z)=\Psi(v)=v^3; \qquad \therefore\ \Psi(F(y))=(F(y))={}^3(F(m))^3.$$

$$\text{and since } F\{y+xf(y)\}=\varepsilon^{m+n\cos v};\ \therefore\ F(y)=\varepsilon^y=\varepsilon^m,$$

putting $x=0$. The nature of the functions expressed by the characters Ψ, f and F being pointed out, Laplace's formula may be applied.

$$u=\Psi(F(y))=(F(m))^3=(\varepsilon^m)^3=\varepsilon^{3m}; \qquad \text{(A)}.$$

$$f(F(y))\frac{d.\Psi(F(y))}{dy}\frac{x}{1}=\cos(\varepsilon^y)\frac{d.\varepsilon^{3y}}{dy}\frac{x}{1}=\cos(\varepsilon^m)\,3\varepsilon^{3m}\,n=3n\,\varepsilon^{3m}\cos\varepsilon^{3m};\text{(B)}.$$

$$\frac{d.\left\{[f(F\quad))]^2\frac{d.\Psi(F(y))}{dy}\right\}}{dy}\frac{x^2}{1.2}=\frac{d.\left\{(\cos(\varepsilon^y))^2\frac{d.\varepsilon^{3y}}{dy}\right\}}{dy}\frac{n^2}{1.2}=$$

$$\left\{-6\varepsilon^{4m}\sin\varepsilon^m+9\varepsilon^{3m}\cos\varepsilon^m\right\}\frac{n^2}{1.2};\qquad (C).$$

&c. &c.

$$\therefore\ u=\Psi(z)=\Psi(v)=v^3=\varepsilon^{3m}=A+B+C+\dots\dots$$

$$\text{or } v^3=\varepsilon^{3m}+3n\varepsilon^{3m}\cos\varepsilon^{3m}+\left\{-6\varepsilon^{4m}\sin\varepsilon^m+9\varepsilon^{3m}\cos\varepsilon^m\right\}\frac{n^2}{1\cdot 2}+\dots$$

a development out of which, in the simplest case, it would be almost impossible to find v^3.

In the equation $v=\varepsilon^{1\cdot 5905201+2\cdot 34\cos v}$ let it be required to find v^3, v, $\cos v$, and the degrees, minutes, &c. corresponding to the arc v.

In $v=\varepsilon^{m+n\cos v}$, put $v=2a\pi+z$; a being 1, 2, 3, 4, &c.

Then $\cos v=\cos z$, and $\therefore\ 2a\pi+z=\varepsilon^{m+n\cos z}$.

$$\therefore\ \log_\varepsilon(2a\pi+z)=m+n\cos z,\ \text{or}\ \frac{\log_\varepsilon(2a\pi+z)-m}{n}=\cos z.$$

$$\text{But } \log_\varepsilon(2a\pi+z)=\log_\varepsilon(2a\pi)+\frac{z}{(2a\pi)}-\frac{z^2}{2(2a\pi)^2}+\frac{z^3}{3(2a\pi)^3}-\dots\dots$$

$$\therefore\ \frac{\log_\varepsilon(2a\pi)-m}{n}+\frac{z}{n(2a\pi)}-\frac{z^2}{2n(2a\pi)^2}+\frac{z^3}{3n(2a\pi)^3}-\frac{z^4}{4n(2a\pi)^4}+\dots=\cos z.$$

It is evident that $\dfrac{\log_\varepsilon(2a\pi)-m}{n}$, must be less than 1, that is $\dfrac{\log_\varepsilon(2a\pi)-1\cdot 5905201}{2\cdot 34}$, must be less than 1.

$\mathrm{Log}_\varepsilon(10\pi)=3\cdot 4473150$; $\log_\varepsilon(12\pi)=3\cdot 6296365$; $\log_\varepsilon(14\pi)=3\cdot 7837872$; hence (12π) may be selected, and $a=6$ is a convenient multiple. Then the equation to be solved is

$$\frac{3\cdot 6296365-1\cdot 5905201}{2\cdot 34}+\frac{z}{12n\pi}-\frac{z^2}{288n\pi^2}+\frac{z^3}{5184n\pi^3}-\dots=\cos z.$$

The following equation may be reduced and the value of z found without difficulty by the dual method of solving equations.

$$\cdot 8714173+\frac{z}{12n\pi}-\frac{z^2}{288n\pi^2}+\frac{z^3}{5184n\pi^3}=1-\frac{z^2}{1.2}+\frac{z^4}{1.2.3.4}-\frac{z^6}{1.2.3.4.5.6}.$$

$$z=\cdot 4\downarrow 2,3,5,\bar{2},5,0,0,8,\ ;\ z=\cdot 4\downarrow 2,3,4,8,4,9,6,2\ :$$

$$z=\cdot 5\downarrow 0,0,2,1,7,6,3,9,\ \&c.=\cdot 5010888764.$$

$$z=\cdot 5010888764=\text{arc of } 28^\circ\ 42'\ 27'';\ \cos z=\cdot 877060106$$

$$v=(2a\pi+z)=(12\pi+z)=38\cdot 2002007=\text{arc of } 4348^\circ\ 42'\ 27''.$$

$$v=38\cdot 2\downarrow 0,0,0,0,0,5,2,6,$$

$$v^3=(38\cdot 2)^3\downarrow 0,0,0,0,1,5,7,8,=55743\cdot 8475.$$

These, and an endless variety of results, cannot be directly and independently obtained by any other known art, formula, or theorem.

$$\begin{array}{ccc} (N) & (D) & (L) \\ \cdot 5010888764 = & \left\{\begin{array}{l} \dot{}4 \downarrow 2,3,4,\underline{8},4,9,\bar{6},2, \\ \dot{}4 \downarrow 2,3,5,\underline{2},\underline{5},\underline{0},\underline{0},\underline{8}, \\ \dot{}4 \downarrow 2,3,5,1,4,9,9,2, \\ \dot{}5 \downarrow 0,0,2,1,7,6,3,9, \\ '1'4'9'9'2\ {}^{2}\underset{4}{\Updownarrow}{}^{1}\ 2,3,5, \\ \quad\&c. \qquad\qquad \&c. \end{array}\right\} & = '69097179_{8}\uparrow \end{array}$$

The developments (D) express the same magnitude under a variety of forms, and are termed dual developments, or numbers, because they have two ultimate representatives, the corresponding natural number (N) on one side, and its logarithm (L) to a known base on the other.

In establishing fundamental principles, in pointing out the laws by which this new art and science are governed, and in comparing new operations with old established systems, the interests of the mere calculator have been in a great measure unavoidably neglected in the larger works. However, although the present work is elementary, it is entirely practical, contains rules, tables easily enlarged, and other subsidiary aids, that will secure accuracy and save both the time and labour of the Astronomer, Navigator, Engineer, Actuary, and calculators in general. The author prepared this note to be published in the analysis of "Dual Arithmetic, a new Art," but an abstract of it was only given, pp. 73 to 81.

NOTE B.

The dual logarithm of $2\cdot = 69314718$,

The hyperbolic logarithm of $2\cdot = \cdot 69314718$

$\dfrac{\downarrow, (1\cdot 1)}{10^8}$ = hyp. log. of $1\cdot 1$ to eight places of decimals; and generally the dual logarithm of any given number n, divided by 10^8 = the hyperbolic logarithm of n, to eight places of decimals. The young student may be deceived by this coincidence and imagine that these systems of logarithms are established by similar processes of reasoning. That such is not the case may be readily shown as follows. Writers on logarithms show by a series of devices that in

the equation $r^x = n$

$$x = \frac{(n-1) - \frac{1}{2}(n-1)^2 + \frac{1}{3}(n-1)^3 - \ldots}{(r-1) - \frac{1}{2}(r-1)^2 + \frac{1}{3}(r-1)^3 - \ldots} \qquad (Z).$$

x being the logarithm of any given number n, to any base r. The expression (Z) cannot be practically applied except in very rare cases.

When the denominator of (Z) is put $= 1$,

that is $(r-1) - \frac{1}{2}(r-1)^2 + \frac{1}{3}(r-1)^3 - \ldots = 1$, then

$r = 2\cdot 7182818 28\ldots$ which is generally represented by ε, and the system is usually termed the hyperbolic system of logarithms.

$$\text{Let } (2{\cdot}71828\ldots)^x = 2{\cdot}$$

then from (Z), $x = (2-1) - \frac{1}{2}(2-1)^2 + \frac{1}{3}(2-1)^3 - \ldots$

$$= 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \ldots$$

$$= {\cdot}69314718 \text{ the hpperbolic log of } 2.$$

To find the sum of the series $1 - \frac{1}{2} + \frac{1}{3} - \ldots$ step by step until we find ·69314718 is a very tedious process. The hyp. logs. of 1·1 ; 1·01 ; 1·001 ; &c. are more readily found by (Z), for let

$$(2{\cdot}71828\ldots)^x = 1{\cdot}1, \text{ then}$$

$$x = (1{\cdot}1 - 1) - \frac{1}{2}(1{\cdot}1 - 1)^2 + \frac{1}{3}(1{\cdot}1 - 1)^3 - \ldots.$$

$$= \frac{1}{10} - \frac{1}{2}\left(\frac{1}{10}\right)^2 + \frac{1}{3}\left(\frac{1}{10}\right)^3 - \ldots.$$

$$= {\cdot}09531018 \text{ the hyp. log of } 1{\cdot}1$$

Now let $(1{\cdot}00000001)^x = 2{\cdot}$ then all the terms of the denominator of (Z) may be neglected except

$$r - 1 = {\cdot}00000001 = \frac{1}{10^8}$$

for $\frac{1}{2}(r-1)^2$; $\frac{1}{3}(r-1)^3$; &c. are very small

In this latter case (Z) gives

$$x = \frac{1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots.}{{\cdot}00000001 + \ldots.}.$$

Hence, the value of x in $(2{\cdot}71828\ldots)^x = 2{\cdot}$ found by (Z) and multiplied by 10^8 is equal to the value of x in $(1{\cdot}00000001)^x = 2{\cdot}$ to eight places of figures, found also by (Z). The same may be said of the application of (Z) to $(2{\cdot}71828\ldots)^x = 1{\cdot}1$ and $(1{\cdot}00000001)^x = 1{\cdot}1$ &c.; but the cases in which (Z) is applicable are very few. These remarks apply to developments with hyperbolic logarithms given in the analysis of "Dual Arithmetic, a new Art," pp. 39 to 46.

Although (Z) indicates that $\frac{\psi_r(n)}{10^8} = \log_\varepsilon n$ true to eight places of decimals for any given number n, yet none of the processes or devices usually employed to apply, establish, or to give to (Z) a more practical form in any way resemble the dual system for finding the logarithm of any given number n, to any base r, by a direct and extremely simple procedure. The young student will avoid being deceived by carefully comparing (Z) and "Analysis," pp. 39 to 46, with the correct dual methods of reduction, Chapter IV., and "Dual Arithmetic, a new Art," pp. 212 to 214.

NOTE C.

☞ The student may be much deceived if strict attention be not given to this note. ☜

A method is given in the analysis, pp. 61 to 72, "Dual Arithmetic,

a new Art," to find dual logarithms by limited tables, and may be made to appear like the dual method; with suitable tables this counterfeit arrangement may be applied to other systems of logarithms, but not without limited tables which the method cannot supply in any case.

The factors of the counterfeit system are $\left\{\begin{array}{llll} 1\cdot9 & 1\cdot8 & 1\cdot7 \ldots\ldots & 1\cdot1 \\ 1\cdot09 & 1\cdot08 & 1\cdot07 \ldots\ldots & 1\cdot01 \\ 1\cdot009 & 1\cdot008 & 1\cdot007 \ldots\ldots & 1\cdot001 \\ & \&c. & \&c. & \end{array}\right.$

The logarithms of these factors being tabulated, and the number decomposed into factors, its logarithm may be found by addition.

For example, $2509621\cdot9 = (1\cdot2)\ (1\cdot04)\ (1\cdot005)\ (1\cdot0004)\ (1\cdot00005)\ (1\cdot000005)\ (1\cdot0000001)\ (1\cdot00000009) \times 2 \times 10^6$; then the dual or any other logarithm of 2509621·9 may be found by adding together the logarithms of the factors (1·2) (1·4) (1·5); &c. taken from tables previously prepared.

The method here alluded to being laborious cannot be relied on; in the analysis, page 72, the method gives 11447298858499266 71, for the dual logarithm of π to seventeen places of figures. In calculating this logarithm by a direct method, it is found to be 114472988584940017. The factors 1·9 1·06 1·003 1·01 &c. have also been employed to approximate to the roots of equations and may deceive a young student or those who possess a smattering of mathematics; but this subject will be discussed in the Author's work on Algebra, and the "Calculus of Functions." In this counterfeit system (1·004) takes the place of $(1\cdot001)^4$; (1·0005) the place of $(1\cdot0001)^5$; &c.

NOTE D.

At page 41, article (53), we promised to explain a method employed to reduce a dual number to a natural one.

The explanation is not intricate, for by common multiplication of algebra, (A) multiplied by (B), gives (C).

$$\begin{array}{ll} \text{(A)} & 1 + a_1x + a_2x^2 + a_3x^3 + a^{1}_{4}x^4 + a_5x^5 + \ldots \\ \text{(B)} & 1 \qquad\quad + b_1x^2 \qquad\qquad + b_2x_4 \qquad\qquad + \ldots \\ \hline \text{(C)} & 1 + a_1x + \left.\begin{array}{c} a_3 \\ 1 \end{array}\right| \begin{array}{c} 1 \\ b_1 \end{array} x^2 + \left.\begin{array}{c} a_3 \\ a_1 \end{array}\right| \begin{array}{c} 1 \\ b_1 \end{array} x^3_3 + \left.\begin{array}{c} a_4 \\ a_2 \\ 1 \end{array}\right| \begin{array}{c} 1 \\ b_1 \\ b_2 \end{array} x^4 + \left.\begin{array}{c} a_5 \\ a_3 \\ a_1 \end{array}\right| \begin{array}{c} 1 \\ b_1 \\ b_2 \end{array} x^5 + \left.\begin{array}{c} a_6 \\ a_4 \\ a_2 \\ a_1 \end{array}\right| \begin{array}{c} 1 \\ b_1 \\ b_2 \\ b_3 \end{array} x^6 + \ldots \end{array}$$

which fully establishes the process. This method of reduction is discussed in "The Art and Science of Dual Arithmetic."

NOTE E.

The student's particular attention is directed to that portion of article 24, page 24, which relates to even roots of negative numbers.

NOTE F.

Without referring to the binomial theorem, the author furnished different methods by which the operative numbers (pp. 21, 35) might be found; only one of these methods was given in the analysis, "Dual

Arithmetic, a new Art," pp. 21 to 24, and that not in the form designed by the author, hence the young student may be misled if an important discrepancy be not pointed out. The operative numbers will not apply to all natural numbers that correspond to consecutive dual numbers. One or two of the numerous examples that might be selected will illustrate this matter.

ASCENDING BRANCH.

$1 \cdot 10356790 = \downarrow 0, 9, 9,$
$1 \cdot 10000000 = \downarrow 1, 0, 0,$
$1 \cdot 10110000 = \downarrow 1, 0, 1,$

$1 \cdot 21392468 = \downarrow 1, 9, 9,$
$1 \cdot 21000000 = \downarrow 2, 0, 0,$
$1 \cdot 21121000 = \downarrow 2, 0, 1,$
&c. &c.

One example from the descending branch will be sufficient.

$\cdot 65998566 = '3\ '9\ '9 \uparrow$
$\cdot 65610000 = '4\ '0\ '0 \uparrow$
$\cdot 65544390 = '4\ '0\ '1 \uparrow$.
&c. &c.

Hence the calculus of differences will only apply to ten consecutive digits of the same rank; and the method of interpolation may or may not point out consecutive dual numbers. The operative dual numbers, or binomial coefficients, may be also found in the following independent manner. Referring to the tabulated form, page 21,

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \ldots \qquad = A;$$

the coefficients being 1, 1, 1, 1, 1, &c., the numbers of the first vertical column in the table.

$$\frac{A}{1-x} = 1 + 2x + 3x^2 + 4x^3 + \ldots \qquad = B;$$

the coefficients being the operative numbers of the second vertical column.

$$\frac{B}{1-x} = 1 + 3x + 6x^2 + 10x^3 + 15x^4 + \ldots = C;$$

the coefficients being the operative numbers of the third vertical column of the table before referred to (pp. 21, 35).

The numbers of the succeeding columns may be found by continuing the division.

NOTE G.

The cosine of an angle is always numerically less than $+1$ or -1; therefore, twice the cosine of an angle must always be numerically less than $+2$ or -2. Now, no whole number or fraction, positive or negative, substituted for x, will render $x + \frac{1}{x}$ numerically less than $+2$ or -2; hence it is absurd to put $2 \cos A = x + \frac{1}{x}$; yet it is on this very absurd supposition that the Theorems of De Moivre, and of Cotes or Vieta, are established, Theorems much admired by Lagrange and Laplace. Dual Arithmetic and the Calculus of Form will clear [illegible]tical science of such imaginary nonsense.

TABLES

OF

ASCENDING & DESCENDING

DUAL LOGARITHMS,

WITH

CORRESPONDING DUAL AND NATURAL NUMBERS.

THE YOUNG DUAL ARITHMETICIAN.

THE Table for the Ascending branch has a range from 1 to 2·99161132, and the Table for the Descending branch ranges from ·29916114 to ·99999999, or to 1. Hence the dual logarithm of 10, ($\downarrow,(10),$) being known, the dual logarithm of any given number may, with much ease, be taken from these abridged tables.

$$
\begin{aligned}
\downarrow,(10) &= 230258509,\text{—}1\\
\downarrow,(100) &= 460517018,\text{—}2\\
\downarrow,(1000) &= 690775527,\text{—}3\\
\downarrow,(10000) &= 921034036,\text{—}4\\
\downarrow,(100000) &= 1151292545,\text{—}5\\
\downarrow,(1000000) &= 1381551054,\text{—}6\\
\downarrow,(10000000) &= 1611809569,\text{—}7\\
\downarrow,(100000000) &= 1842068072,\text{—}8\\
\downarrow,(1000000000) &= 2072326581,\text{—}9
\end{aligned}
$$

EXAMPLES.

1. Find the dual logarithm of ·86194541, ·00086194541, and 86194541.

$\downarrow,(\cdot86194541) =\ '14856338$ Tab. II.
$\downarrow,(10^8) = 1842068072,$

$\downarrow,(86194541.) = 1827211734,$

$'14856338$
Negative $\downarrow,(10^3)$ $'690775527$

$\downarrow,(\cdot00086194541) =\ '705631865$

2. Find the dual logarithm of 1·378201, 1378·201; and ·01378201.

The dual log. of 1·378201 from Table I. is
32077903,
$\downarrow,(10^3) = 690775527,$

$\downarrow,(1378\cdot201) = 722853430,$

Negative ↓,(10^2) = '460517018
↓,(1·378201) = 32077903,

↓,(·01378201) = '428439115

3. Required the dual number and logarithm of 1·865 and the dual logarithm of 1865.

		1·86500000 given number.		
↓ 6,5,1,0,0,0,0,0,	=	1·863|7903|4	= 62261223,	Tab. I.
6,4,8,8,5,	↓^{4}6,	1|1182|8	64885,	
		2|8		
			62326108,	
↓ 6,5,1,6,4,8,8,5,		186490890	take from given No.	
		··· † 9110	diff.	
	↓^{5}4,	7460		
		1650		
	↓^{6}8,	1491		
		159		
	↓^{7}8,	150		
	↓^{8}5,	9		

∴ ↓,(1·865) = 623261O8,
↓,(10^3) = 690775527,

753101635, = ↓,(1865.)

4. Find the dual number and logarithm of ·78539816 and the dual logarithm of ·078539816.

'2'3'0'0'0'0'0'0 ↑	=	·7859|4219	= '24087205	Tab. II.
'6'9'2'4'2	↓^{4}6,	4.7156 −	'69242	
		· 12 +		
'2'3'0'6'9'2'4'2 ↑				
		·78547074	'24156447	
Given		·78539816		
		7258		
	↓^{5}9,	7069		
		1|89		
	↓^{6}2,	1|57		
		3|2		
	↓^{7}4,	3|1		
		1|		
	↓^{8}2,	1|		

$$\begin{array}{lrcl} & \therefore \downarrow,(\cdot 78539816) & = & '24156447 \\ \text{Negative} & \downarrow,(10) & & '230258509 \\ \hline & \downarrow,(\cdot 078539816) & = & '254414956 \end{array}$$

5. Find the dual logarithm, ($\downarrow,$), and common number answering to $\downarrow,1,2,3,4,5,6,7,8,$

$$\begin{array}{rcrl} \downarrow 1,2,3,0,0,0,0,0, & = & \downarrow 11820934, & \text{Tab. I.} \\ 4,5,6,7,8, & = & 45678, & \\ \hline \therefore \ \downarrow 1,2,3,4,5,6,7,8, & = & \downarrow 11866612, & \end{array}$$

Again, Tab. I. $\downarrow 1,2,3,0,0,0,0,0, = 1{\cdot}125\,4797\,0$

$$\begin{array}{r|r|rl} 1{\cdot}125 & 4797 & 0 & \\ & 4501 & 9 & \downarrow^4 4, \\ & & 7 & \\ \hline 1{\cdot}1259 & 2996 & & \\ \cdots\dagger & 5629 & & \uparrow^5 5, \\ & 675 & & \downarrow^6 6, \\ & 78 & & \downarrow^7 7, \\ & 9 & & \downarrow^8 8, \end{array}$$

Common number $1{\cdot}2599387$

6. What dual number and common number corresponds to the dual logarithm 34276352,

$$\begin{array}{rcrl} 34276352, & & & \\ \downarrow 34267871, & = & \downarrow 3,5,7,0,0,0,0,0, & \text{Tab. I.} \\ \hline 8481, & & 8,4,8,1, & \\ & & \downarrow 3,5,7,0,8,4,8,1, & \end{array}$$

$$\begin{array}{rcl} \therefore \ \downarrow 3,5,7,0,8,4,8,1, & = & 34276352, \\ \downarrow 3,5,7,0,0,0,0,0, & = & 1{\cdot}4087\,|\,9614 \\ & & \cdots\dagger 1\,|\,1270 \quad \downarrow^5 8, \\ & & 563 \quad \downarrow^6 4, \\ & & 113 \quad \downarrow^7 8, \\ & & 1 \quad \downarrow^8 1, \end{array}$$

Required common number $1{\cdot}40891561$

7. Required both the dual and common number corresponding to the dual logarithm 1177778277.

When the given logarithm is too great to be found in the table, mark off seven figures and take the next less multiple of 23 contained in what remains, thus the power of 10 involved may be determined by mere inspection.

Thus 117 contains 23 five times, but not six times.

1177778277,
$\downarrow$,(10^5) = 1151292545,

26485732,
$\downarrow$,2,7,4,0,0,0,0,0, = 26427067, = 1·30117951 Tab. I.
5,8,6,6,5,
58865,

$\downarrow$ 2,7,4,5,8,6,6,5,

∴ 10^5 $\downarrow$ 2,7,4,5,8,6,6,5, = $\downarrow$1177778277,

1·30117951
65059
13

13018|3023
···†1|0415
781
78
7

1·30194304

∴ $\downarrow$,(130194·304) = 1177778277,

8. Required both the dual and common numbers corresponding to the dual logarithm '83600000;

·43344921 = '7'9'8'0'0'0'0'0 $\uparrow$ = '93598067; Tab. II.

'83600000
'83598067

'00001933

·43344|921
··† |433—
390—
13—
1—

837

·43344084

∴ ·43344084 = '7'9'8'0'1'9'3'3 $\uparrow$ = '83600000

9. Required both the dual and common numbers, corresponding to the dual logarithm '1177693322.

·76799335='2'5'3'0'0'0'0'0 ↑='26397423 ; Tab. II.

'1177693322 given.
↓,(10^5) = '1151292545 when made negative.

'26400777
'26397423 from Tab. II.

'3354

∴ '2'5'3'0'3'3'5'4 ↑ $_{\frac{1}{(10)^5}}$ = '1177693322$_8$↑

·76799|335
··† 2|304—
|230—
38 —
3 —

2575

·76796760

∴ ↓, (·000007679676) = '1177693322

Numbers and their corresponding dual logarithms may be found by merely employing a part of Table I. from 1 to 2, but in doing so, ↓, (2)=69314718, as well as ↓, (10)=230258509, have to be involved.

EXAMPLE.

10. Let it be required to find the dual logarithm and common number answering to $10^3 \times 2 \times$ ↓ 3,3,7,4,5,6,7,3, = 2000 ↓ 3,3,7,4,5,6,7,3.

From Table I. the following line may be taken:—

1·38095878= ↓ 3,3,7,0,0,0,0,0,=32277805,
to which add 4,5,6,7,3,= 45673

↓.(1·38095878)= 32323478,
↓,(10^3)=690775530,
↓,(2) = 69314718,

792413726,

```
1·380|9587|8‿4
     |5523|8
     |    |8
---------------
1·3815|1124
....  |6908‿5
      | 829‿6
      |  97‿7
      |   4‿8
---------------
1·38158962
         2
---------------
2763·17924 Natural Number.
```

Hence 2763·17924 is the natural number corresponding to the dual number $10^3 \times 2 \downarrow 3,3,7,4,5,6,7,3$, and 792413726, is the ultimate representative number in the eighth position and may be employed as the logarithm of 2763·17924; written, $\downarrow$,(2763·17924)=792413726.

When results are only required true to five or six places of figures, to extract corresponding numbers from Table I. is a thoroughly easy matter.

The above example, to this extent, will stand as follows:—

```
1·380|959..   = ↓3,3,7,0,0,0,.. = 322778 ..
...  |552‿4          4,5,7,..       457 ..
     | 69‿5                       ---------
     |  9‿7                       323235 ..
------------                     6907755 ..
1·381589                          693147 ..
       2                         ----------
--------                         7924137
2763·178
```

TABLE I.

ASCENDING BRANCH.

ABRIDGEMENT

OF

BYRNE'S TABLE OF DUAL NUMBERS,

WITH

CORRESPONDING NATURAL NUMBERS

AND

THEIR DUAL LOGARITHMS.

NATURAL NUMBERS.	DUAL NUMBERS.	DUAL LOGARITHMS.
N. Nos.	D. Nos.	D. Logs.
1·00000000	↓0,0,0,0,0,0,0,0,	00000000,
1·00100000	↓0,0,1,0,0,0,0,0,	99950,
1·00200100	↓0,0,2,0,0,0,0,0,	199900,
1·00300300	↓0,0,3,0,0,0,0,0,	299850,
1·00400600	↓0,0,4,0,0,0,0,0,	399800,
1·00501001	↓0,0,5,0,0,0,0,0,	499750,
1·00601502	↓0,0,6,0,0,0,0,0,	599702,
1·00702104	↓0,0,7,0,0,0,0,0,	699652,
1·00802806	↓0,0,8,0,0,0,0,0,	799602,
1·00903608	↓0,0,9,0,0,0,0,0,	899552,
1·01000000	↓0,1,0,0,0,0,0,0,	995033,
1·01101000	↓0,1,1,0,0,0,0,0,	1094983,
1·01202101	↓0,1,2,0,0,0,0,0,	1194933,
1·01303303	↓0,1,3,0,0,0,0,0,	1294883,
1·01404606	↓0,1,4,0,0,0,0,0,	1394833,
1·01506011	↓0,1,5,0,0,0,0,0,	1494783,
1·01607517	↓0,1,6,0,0,0,0,0,	1594735,
1·01709125	↓0,1,7,0,0,0,0,0,	1694655,
1·01810834	↓0,1,8,0,0,0,0,0,	1794635,
1·01912644	↓0,1,9,0,0,0,0,0,	1894585,
1·02010000	↓0,2,0,0,0,0,0,0,	1990066,
1·02112010	↓0,2,1,0,0,0,0,0,	2090016,
1·02214122	↓0,2,2,0,0,0,0,0,	2189966,
1·02316336	↓0,2,3,0,0,0,0,0,	2289916,
1·02418652	↓0,2,4,0,0,0,0,0,	2389866,
1·02521071	↓0,2,5,0,0,0,0,0,	2489816,
1·02623592	↓0,2,6,0,0,0,0,0,	2589768,
1·02726216	↓0,2,7,0,0,0,0,0,	2689718,
1·02828942	↓0,2,8,0,0,0,0,0,	2789668,
1·02931771	↓0,2,9,0,0,0,0,0,	2889618,
1·03030100	↓0,3,0,0,0,0,0,0,	2985099,
1·03133130	↓0,3,1,0,0,0,0,0,	3085049,
1·03236263	↓0,3,2,0,0,0,0,0,	3184999,
1·03339499	↓0,3,3,0,0,0,0,0,	3284949,
1·03442838	↓0,3,4,0,0,0,0,0,	3384899,
1·03546281	↓0,3,5,0,0,0,0,0,	3484849,
1·03649827	↓0,3,6,0,0,0,0,0,	3584801,
1·03753478	↓0,3,7,0,0,0,0,0,	3684751,
1·03857230	↓0,3,8,0,0,0,0,0,	3784701,
1·03961088	↓0,3,9,0,0,0,0,0,	3884651,
1·04060401	↓0,4,0,0,0,0,0,0,	3980132,
1·04164461	↓0,4,1,0,0,0,0,0,	4080082,
1·04268626	↓0,4,2,0,0,0,0,0,	4180032,
1·04372894	↓0,4,3,0,0,0,0,0,	4279982,
1·04477267	↓0,4,4,0,0,0,0,0,	4379932,
1·04581745	↓0,4,5,0,0,0,0,0,	4479882,
1·04686326	↓0,4,6,0,0,0,0,0,	4579834,
1·04791013	↓0,4,7,0,0,0,0,0,	4679784,
1·04895804	↓0,4,8,0,0,0,0,0,	4779734,
1·05000700	↓0,4,9,0,0,0,0,0,	4879684,
1·05101005	↓0,5,0,0,0,0,0,0,	4975165,

2 = 69314718 10 = 230258509

N. Nos.	D. Nos.	D. Logs.
1·05206106	↓ 0,5,1,0,0,0,0,0,	5075115,
1·05311312	↓ 0,5,2,0,0,0,0,0,	5175065,
1·05416623	↓ 0,5,3,0,0,0,0,0,	5275015,
1·05522040	↓ 0,5,4,0,0,0,0,0,	5374965,
1·05627562	↓ 0,5,5,0,0,0,0,0,	5474915,
1·05733190	↓ 0,5,6,0,0,0,0,0,	5574867,
1·05838923	↓ 0,5,7,0,0,0,0,0,	5674817,
1·05944762	↓ 0,5,8,0,0,0,0,0,	5774765,
1·06050707	↓ 0,5,9,0,0,0,0,0,	5874717,
1·06152015	↓ 0,6,0,0,0,0,0,0,	5970198,
1·06258167	↓ 0,6,1,0,0,0,0,0,	6070148,
1·06364425	↓ 0,6,2,0,0,0,0,0,	6170098,
1·06470789	↓ 0,6,3,0,0,0,0,0,	6270048,
1·06577260	↓ 0,6,4,0,0,0,0,0,	6369998,
1·06683838	↓ 0,6,5,0,0,0,0,0,	6469948,
1·06790521	↓ 0,6,6,0,0,0,0,0,	6569900,
1·06897312	↓ 0,6,7,0,0,0,0,0,	6669850,
1·07004209	↓ 0,6,8,0,0,0,0,0,	6769800,
1·07111213	↓ 0,6,9,0,0,0,0,0,	6869740,
1·07213535	↓ 0,7,0,0,0,0,0,0,	6965231,
1·07320749	↓ 0,7,1,0,0,0,0,0,	7065181,
1·07428069	↓ 0,7,2,0,0,0,0,0,	7165131,
1·07535498	↓ 0,7,3,0,0,0,0,0,	7265081,
1·07643032	↓ 0,7,4,0,0,0,0,0,	7365031,
1·07750676	↓ 0,7,5,0,0,0,0,0,	7464981,
1·07858426	↓ 0,7,6,0,0,0,0,0,	7564933,
1·07966285	↓ 0,7,7,0,0,0,0,0,	7664883,
1·08074251	↓ 0,7,8,0,0,0,0,0,	7764833,
1·08182326	↓ 0,7,9,0,0,0,0,0,	7864783,
1·08285671	↓ 0,8,0,0,0,0,0,0,	7960264,
1·08393957	↓ 0,8,1,0,0,0,0,0,	8060214,
1·08502350	↓ 0,8,2,0,0,0,0,0,	8160164,
1·08610853	↓ 0,8,3,0,0,0,0,0,	8260114,
1·08719464	↓ 0,8,4,0,0,0,0,0,	8360064,
1·08828183	↓ 0,8,5,0,0,0,0,0,	8460014,
1·08937011	↓ 0,8,6,0,0,0,0,0,	8559966,
1·09045949	↓ 0,8,7,0,0,0,0,0,	8659916,
1·09154994	↓ 0,8,8,0,0,0,0,0,	8759866,
1·09264149	↓ 0,8,9,0,0,0,0,0,	8859816,
1·09368527	↓ 0,9,0,0,0,0,0,0,	8955297,
1·09477896	↓ 0,9,1,0,0,0,0,0,	9055247,
1·09587374	↓ 0,9,2,0,0,0,0,0,	9155197,
1·09696961	↓ 0,9,3,0,0,0,0,0,	9255147,
1·09806658	↓ 0,9,4,0,0,0,0,0,	9355097,
1·09916465	↓ 0,9,5,0,0,0,0,0,	9455047,
1·10026381	↓ 0,9,6,0,0,0,0,0,	9554999,
1·10136407	↓ 0,9,7,0,0,0,0,0,	9654949,
1·10246543	↓ 0,9,8,0,0,0,0,0,	9754899,
1·10356790	↓ 0,9,9,0,0,0,0,0,	9854849,
1·10000000	↓ 1,0,0,0,0,0,0,0,	9531018,

2= 69314718	10=230258509
4=138629436	100=460517018
8=207944154	1000=690775527

N. Nos.	D. Nos.	D. Logs.
1·10110000	↓ 1,0,1,0,0,0,0,0,	9630968,
1·10220110	↓ 1,0,2,0,0,0,0,0,	9730918,
1·10330330	↓ 1,0,3,0,0,0,0,0,	9830868,
1·10440660	↓ 1,0,4,0,0,0,0,0,	9930818,
1·10551100	↓ 1,0,5,0,0,0,0,0,	10030768,
1·10661651	↓ 1,0,6,0,0,0,0,0,	10130720,
1·10772313	↓ 1,0,7,0,0,0,0,0,	10230670,
1·10883086	↓ 1,0,8,0,0,0,0,0,	10330628,
1·10993969	↓ 1,0,9,0,0,0,0,0,	10430570,
1·11100000	↓ 1,1,0,0,0,0,0,0,	10526051,
1·11211000	↓ 1,1,1,0,0,0,0,0,	10626001,
1·11322211	↓ 1,1,2,0,0,0,0,0,	10725951,
1·11433533	↓ 1,1,3,0,0,0,0,0,	10825901,
1·11545067	↓ 1,1,4,0,0,0,0,0,	10925851,
1·11656612	↓ 1,1,5,0,0,0,0,0,	11025801,
1·11768269	↓ 1,1,6,0,0,0,0,0,	11125753,
1·11880037	↓ 1,1,7,0,0,0,0,0,	11225703,
1·11991917	↓ 1,1,8,0,0,0,0,0,	11325653,
1·12103909	↓ 1,1,9,0,0,0,0,0,	11425603,
1·12211000	↓ 1,2,0,0,0,0,0,0,	11521084,
1·12323211	↓ 1,2,1,0,0,0,0,0,	11621034,
1·12435534	↓ 1,2,2,0,0,0,0,0,	11720984,
1·12547970	↓ 1,2,3,0,0,0,0,0,	11820934,
1·12660517	↓ 1,2,4,0,0,0,0,0,	11920884,
1·12773178	↓ 1,2,5,0,0,0,0,0,	12020834,
1·12885951	↓ 1,2,6,0,0,0,0,0,	12120786,
1·12998837	↓ 1,2,7,0,0,0,0,0,	12220736,
1·13111836	↓ 1,2,8,0,0,0,0,0,	12320686,
1·13224948	↓ 1,2,9,0,0,0,0,0,	12420636,
1·13333110	↓ 1,3,0,0,0,0,0,0,	12516117,
1·13446443	↓ 1,3,1,0,0,0,0,0,	12616067,
1·13559889	↓ 1,3,2,0,0,0,0,0,	12716017,
1·13673449	↓ 1,3,3,0,0,0,0,0,	12815967,
1·13787122	↓ 1,3,4,0,0,0,0,0,	12915917,
1·13900909	↓ 1,3,5,0,0,0,0,0,	13015867,
1·14014810	↓ 1,3,6,0,0,0,0,0,	13115819,
1·14128825	↓ 1,3,7,0,0,0,0,0,	13215769,
1·14242954	↓ 1,3,8,0,0,0,0,0,	13315719,
1·14357197	↓ 1,3,9,0,0,0,0,0,	13415669,
1·14466441	↓ 1,4,0,0,0,0,0,0,	13511150,
1·14580907	↓ 1,4,1,0,0,0,0,0,	13611100,
1·14695488	↓ 1,4,2,0,0,0,0,0,	13711050,
1·14810183	↓ 1,4,3,0,0,0,0,0,	13811000,
1·14924993	↓ 1,4,4,0,0,0,0,0,	13910950,
1·15039918	↓ 1,4,5,0,0,0,0,0,	14010900,
1·15154958	↓ 1,4,6,0,0,0,0,0,	14110852,
1·15270113	↓ 1,4,7,0,0,0,0,0,	14210802,
1·15385383	↓ 1,4,8,0,0,0,0,0,	14310752,
1·15500768	↓ 1,4,9,0,0,0,0,0,	14410702,
1·15611106	↓ 1,5,0,0,0,0,0,0,	14506183,

2 = 69314718	10 = 230258509
4 = 138629436	100 = 460517018
8 = 207944154	1000 = 690775527

N. Nos.	D. Nos.	D. Logs.
1·15726717	↓ 1,5,1,0,0,0,0,0,	14606133,
1·15842444	↓ 1,5,2,0,0,0,0,0,	14706083,
1·15958286	↓ 1,5,3,0,0,0,0,0,	14806033,
1·16074244	↓ 1,5,4,0,0,0,0,0,	14905983,
1·16190319	↓ 1,5,5,0,0,0,0,0,	15005933,
1·16306509	↓ 1,5,6,0,0,0,0,0,	15105885,
1·16422815	↓ 1,5,7,0,0,0,0,0,	15205835,
1·16539238	↓ 1,5,8,0,0,0,0,0,	15305785,
1·16655777	↓ 1,5,9,0,0,0,0,0,	15405735,
1·16767217	↓ 1,6,0,0,0,0,0,0,	15501216,
1·16883984	↓ 1,6,1,0,0,0,0,0,	15601166,
1·17000868	↓ 1,6,2,0,0,0,0,0,	15701116,
1·17117869	↓ 1,6,3,0,0,0,0,0,	15801066,
1·17234987	↓ 1,6,4,0,0,0,0,0,	15901016,
1·17352222	↓ 1,6,5,0,0,0,0,0,	16000966,
1·17469574	↓ 1,6,6,0,0,0,0,0,	16100918,
1·17587044	↓ 1,6,7,0,0,0,0,0,	16200868,
1·17704631	↓ 1,6,8,0,0,0,0,0,	16300818,
1·17822336	↓ 1,6,9,0,0,0,0,0,	16400768,
1·17934889	↓ 1,7,0,0,0,0,0,0,	16496249,
1·18052824	↓ 1,7,1,0,0,0,0,0,	16596199,
1·18170877	↓ 1,7,2,0,0,0,0,0,	16696149,
1·18289048	↓ 1,7,3,0,0,0,0,0,	16796099,
1·18407337	↓ 1,7,4,0,0,0,0,0,	16896049,
1·18525774	↓ 1,7,5,0,0,0,0,0,	16995999,
1·18644270	↓ 1,7,6,0,0,0,0,0,	17095951,
1·18762914	↓ 1,7,7,0,0,0,0,0,	17195901,
1·18881677	↓ 1,7,8,0,0,0,0,0,	17295851,
1·19000559	↓ 1,7,9,0,0,0,0,0,	17395799,
1·19114238	↓ 1,8,0,0,0,0,0,0,	17491264,
1·19233352	↓ 1,8,1,0,0,0,0,0,	17591214,
1·19352585	↓ 1,8,2,0,0,0,0,0,	17691164,
1·19471938	↓ 1,8,3,0,0,0,0,0,	17791110,
1·19591410	↓ 1,8,4,0,0,0,0,0,	17891060,
1·19711001	↓ 1,8,5,0,0,0,0,0,	17991010,
1·19830712	↓ 1,8,6,0,0,0,0,0,	18090962,
1·19950543	↓ 1,8,7,0,0,0,0,0,	18190912,
1·20070494	↓ 1,8,8,0,0,0,0,0,	18290862,
1·20190564	↓ 1,8,9,0,0,0,0,0,	18390812,
1·20305380	↓ 1,9,0,0,0,0,0,0,	18486315,
1·20425685	↓ 1,9,1,0,0,0,0,0,	18586265,
1·20546111	↓ 1,9,2,0,0,0,0,0,	18686215,
1·20666657	↓ 1,9,3,0,0,0,0,0,	18786165,
1·20787323	↓ 1,9,4,0,0,0,0,0,	18886115,
1·20908110	↓ 1,9,5,0,0,0,0,0,	18986065,
1·21029018	↓ 1,9,6,0,0,0,0,0,	19086017,
1·21150047	↓ 1,9,7,0,0,0,0,0,	19185967,
1·21271197	↓ 1,9,8,0,0,0,0,0,	19285917,
1·21392468	↓ 1,9,9,0,0,0,0,0,	19385867,
1·21000000	↓ 2,0,0,0,0,0,0,0,	19062036,

2= 69314718 10=230258509
4=138629436 100=460517018
8=207944154 1000=690775527

N. Nos.	D. Nos.	D. Logs.
1·21121000	↓2,0,1,0,0,0,0,0,	19161986,
1·21242121	↓2,0,2,0,0,0,0,0,	19261936,
1·21363363	↓2,0,3,0,0,0,0,0,	19361886,
1·21484726	↓2,0,4,0,0,0,0,0,	19461836,
1·21606211	↓2,0,5,0,0,0,0,0,	19561788,
1·21727817	↓2,0,6,0,0,0,0,0,	19661738,
1·21849545	↓2,0,7,0,0,0,0,0,	19761688,
1·21971395	↓2,0,8,0,0,0,0,0,	19861638,
1·22093366	↓2,0,9,0,0,0,0,0,	19961588,
1·22210000	↓2,1,0,0,0,0,0,0,	20057069,
1·22332210	↓2,1,1,0,0,0,0,0,	20157019,
1·22454542	↓2,1,2,0,0,0,0,0,	20256969,
1·22576996	↓2,1,3,0,0,0,0,0,	20356919,
1·22699573	↓2,1,4,0,0,0,0,0,	20456869,
1·22822273	↓2,1,5,0,0,0,0,0,	20556819,
1·22945095	↓2,1,6,0,0,0,0,0,	20656769,
1·23068040	↓2,1,7,0,0,0,0,0,	20756721,
1·23191108	↓2,1,8,0,0,0,0,0,	20856671,
1·23214299	↓2,1,9,0,0,0,0,0,	20956621,
1·23432100	↓2,2,0,0,0,0,0,0,	21052102,
1·23555532	↓2,2,1,0,0,0,0,0,	21152052,
1·23679088	↓2,2,2,0,0,0,0,0,	21252002,
1·23802767	↓2,2,3,0,0,0,0,0,	21351952,
1·23926570	↓2,2,4,0,0,0,0,0,	21451902,
1·24050497	↓2,2,5,0,0,0,0,0,	21551852,
1·24174547	↓2,2,6,0,0,0,0,0,	21651804,
1·24298722	↓2,2,7,0,0,0,0,0,	21751754,
1·24423021	↓2,2,8,0,0,0,0,0,	21851704,
1·24547444	↓2,2,9,0,0,0,0,0,	21951654,
1·24666421	↓2,3,0,0,0,0,0,0,	22047135,
1·24791087	↓2,3,1,0,0,0,0,0,	22147085,
1·24915878	↓2,3,2,0,0,0,0,0,	22247035,
1·25040794	↓2,3,3,0,0,0,0,0,	22346985,
1·25165835	↓2,3,4,0,0,0,0,0,	22446935,
1·25291001	↓2,3,5,0,0,0,0,0,	22546885,
1·25416292	↓2,3,6,0,0,0,0,0,	22646835,
1·25541708	↓2,3,7,0,0,0,0,0,	22746785,
1·25667250	↓2,3,8,0,0,0,0,0,	22846735,
1·25792917	↓2,3,9,0,0,0,0,0,	22946687,
1·25913084	↓2,4,0,0,0,0,0,0,	23042168,
1·26038998	↓2,4,1,0,0,0,0,0,	23142118,
1·26165037	↓2,4,2,0,0,0,0,0,	23242068,
1·26291202	↓2,4,3,0,0,0,0,0,	23342018,
1·26417493	↓2,4,4,0,0,0,0,0,	23441968,
1·26543911	↓2,4,5,0,0,0,0,0,	23541918,
1·26670455	↓2,4,6,0,0,0,0,0,	23641868,
1·26797125	↓2,4,7,0,0,0,0,0,	23741818,
1·26923922	↓2,4,8,0,0,0,0,0,	23841764,
1·27050846	↓2,4,9,0,0,0,0,0,	23941714,
1·27172216	↓2,5,0,0,0,0,0,0,	24037201,

2 = 69314718 10 = 230258509
4 = 138629436 100 = 460517018
8 = 207944154 1000 = 690775527

N. Nos.	D. Nos.	D. Logs.
1·27299388	↓2,5,1,0,0,0,0,0,	24137151,
1·27426687	↓2,5,2,0,0,0,0,0,	24237101,
1·27554114	↓2,5,3,0,0,0,0,0,	24337051,
1·27681668	↓2,5,4,0,0,0,0,0,	24437001,
1·27809350	↓2,5,5,0,0,0,0,0,	24536951,
1·27937159	↓2,5,6,0,0,0,0,0,	24636903,
1·28065096	↓2,5,7,0,0,0,0,0,	24736853,
1·28193161	↓2,5,8,0,0,0,0,0,	24836803,
1·28321354	↓2,5,9,0,0,0,0,0,	24936753,
1·28443938	↓2,6,0,0,0,0,0,0,	25032234,
1·28572382	↓2,6,1,0,0,0,0,0,	25132184,
1·28700954	↓2,6,2,0,0,0,0,0,	25232134,
1·28829655	↓2,6,3,0,0,0,0,0,	25332084,
1·28958485	↓2,6,4,0,0,0,0,0,	25432034,
1·29087443	↓2,6,5,0,0,0,0,0,	25531984,
1·29216530	↓2,6,6,0,0,0,0,0,	25631936,
1·29345747	↓2,6,7,0,0,0,0,0,	25731886,
1·29475093	↓2,6,8,0,0,0,0,0,	25831836,
1·29604568	↓2,6,9,0,0,0,0,0,	25531786,
1·29728377	↓2,7,0,0,0,0,0,0,	26027267,
1·29858105	↓2,7,1,0,0,0,0,0,	26127217,
1·29987963	↓2,7,2,0,0,0,0,0,	26227167,
1·30117951	↓2,7,3,0,0,0,0,0,	26327117,
1·30248069	↓2,7,4,0,0,0,0,0,	26427067,
1·30378317	↓2,7,5,0,0,0,0,0,	26527017,
1·30508695	↓2,7,6,0,0,0,0,0,	26626967,
1·30639204	↓2,7,7,0,0,0,0,0,	26726917,
1·30769843	↓2,7,8,0,0,0,0,0,	26826869,
1·30900613	↓2,7,9,0,0,0,0,0,	26926819,
1·31025662	↓2,8,0,0,0,0,0,0,	27022300,
1·31156688	↓2,8,1,0,0,0,0,0,	27122250,
1·31287845	↓2,8,2,0,0,0,0,0,	27222200,
1·31419133	↓2,8,3,0,0,0,0,0,	27322150,
1·31550552	↓2,8,4,0,0,0,0,0,	27422100,
1·31682103	↓2,8,5,0,0,0,0,0,	27522050,
1·31813785	↓2,8,6,0,0,0,0,0,	27622002,
1·31945599	↓2,8,7,0,0,0,0,0,	27721952,
1·32077545	↓2,8,8,0,0,0,0,0,	27821902,
1·32209623	↓2,8,9,0,0,0,0,0,	27921852,
1·32335918	↓2,9,0,0,0,0,0,0,	28017333,
1·32468254	↓2,9,1,0,0,0,0,0,	28117283,
1·32600722	↓2,9,2,0,0,0,0,0,	28217233.
1·32733323	↓2,9,3,0,0,0,0,0,	28317183,
1·32866056	↓2,9,4,0,0,0,0,0,	28417133,
1·32998922	↓2,9,5,0,0,0,0,0,	28517083,
1·33131921	↓2,9,6,0,0,0,0,0,	28617035,
1·33265053	↓2,9,7,0,0,0,0,0,	28716985,
1·33398318	↓2,9,8,0,0,0,0,0,	28816935,
1·33531716	↓2,9,9,0,0,0,0,0,	28916885,
1·33100000	↓3,0,0,0,0,0,0,0,0,	28593054,

2= 69314718	10=230258509
4=138629436	100=460517018
8=207944154	1000=690775527

N. Nos.	D. Nos.	D. Logs.
1·33233100	↓3,0,1,0,0,0,0,0,	28693004,
1·33366333	↓3,0,2,0,0,0,0,0,	28792954,
1·33499699	↓3,0,3,0,0,0,0,0,	28892904,
1·33633199	↓3,0,4,0,0,0,0,0,	28992854,
1·33766832	↓3,0,5,0,0,0,0,0,	29092804,
1·33900599	↓3,0,6,0,0,0,0,0,	29192756,
1·34034499	↓3,0,7,0,0,0,0,0,	29292706,
1·34168533	↓3,0,8,0,0,0,0,0,	29392655,
1·34302702	↓3,0,9,0,0,0,0,0,	29492605,
1·34431000	↓3,1,0,0,0,0,0,0,	29588087,
1·34565431	↓3,1,1,0,0,0,0,0,	29688037,
1·34699996	↓3,1,2,0,0,0,0,0,	29787987,
1·34834696	↓3,1,3,0,0,0,0,0,	29887937,
1·34969531	↓3,1,4,0,0,0,0,0,	29987887,
1·35104501	↓3,1,5,0,0,0,0,0,	30087837,
1·35239606	↓3,1,6,0,0,0,0,0,	30187789,
1·35374846	↓3,1,7,0,0,0,0,0,	30287739,
1·35510221	↓3,1,8,0,0,0,0,0,	30387689,
1·35645731	↓3,1,9,0,0,0,0,0,	30487639,
1·35775310	↓3,2,0,0,0,0,0,0,	30583120,
1·35911085	↓3,2,1,0,0,0,0,0,	30683070,
1·36046996	↓3,2,2,0,0,0,0,0,	30783020,
1·36183043	↓3,2,3,0,0,0,0,0,	30882970,
1·36319226	↓3,2,4,0,0,0,0,0,	30982920,
1·36455545	↓3,2,5,0,0,0,0,0,	31082870,
1·36592001	↓3,2,6,0,0,0,0,0,	31182820,
1·36728593	↓3,2,7,0,0,0,0,0,	31282772,
1·36865322	↓3,2,8,0,0,0,0,0,	31382722,
1·37002187	↓3,2,9,0,0,0,0,0,	31382672,
1·37133063	↓3,3,0,0,0,0,0,0,	31578153,
1·37270196	↓3,3,1,0,0,0,0,0,	31678103,
1·37407466	↓3,3,2,0,0,0,0,0,	31778053,
1·37544873	↓3,3,3,0,0,0,0,0,	31878003,
1·37682418	↓3,3,4,0,0,0,0,0,	31977953,
1·37820100	↓3,3,5,0,0,0,0,0,	32077903,
1·37957920	↓3,3,6,0,0,0,0,0,	32177853,
1·38095878	↓3,3,7,0,0,0,0,0,	32277805,
1·38233974	↓3,3,8,0,0,0,0,0,	32377755,
1·38372208	↓3,3,9,0,0,0,0,0,	32477705,
1·38504393	↓3,4,0,0,0,0,0,0,	32573186,
1·38642897	↓3,4,1,0,0,0,0,0,	32673136,
1·38781540	↓3,4,2,0,0,0,0,0,	32773086,
1·38920322	↓3,4,3,0,0,0,0,0,	32873036,
1·39059242	↓3,4,4,0,0,0,0,0,	32972986,
1·39198301	↓3,4,5,0,0,0,0,0,	33072936,
1·39337500	↓3,4,6,0,0,0,0,0,	33172888,
1·39576839	↓3,4,7,0,0,0,0,0,	33272838,
1·39616314	↓3,4,8,0,0,0,0,0,	33372788,
1·39755930	↓3,4,9,0,0,0,0,0,	33472738,
1·39889438	↓3,5,0,0,0,0,0,0,	33568219,

2 = 69314718 10 = 230258509
4 = 138629436 100 = 460517018
8 = 207944154 1000 = 690775527

N. Nos.	D. Nos.	D. Logs.
1·40029327	↓ 3,5,1,0,0,0,0,0,	33668169,
1·40169356	↓ 3,5,2,0,0,0,0,0,	33768119,
1·40309525	↓ 3,5,3,0,0,0,0,0,	33868069,
1·40449835	↓ 3,5,4,0,0,0,0,0,	33968019,
1·40590285	↓ 3,5,5,0,0,0,0,0,	34067969,
1·40730875	↓ 3,5,6,0,0,0,0,0,	34167921,
1·40871606	↓ 3,5,7,0,0,0,0,0,	34267871,
1·41012478	↓ 3,5,8,0,0,0,0,0,	34367821,
1·41153490	↓ 3,5,9,0,0,0,0,0,	34467772,
1·41288332	↓ 3,6,0,0,0,0,0,0,	34563252,
1·41429620	↓ 3,6,1,0,0,0,0,0,	34663202,
1·41571050	↓ 3,6,2,0,0,0,0,0,	34763152,
1·41712621	↓ 3,6,3,0,0,0,0,0,	34863102,
1·41854333	↓ 3,6,4,0,0,0,0,0,	34963052,
1·41996187	↓ 3,6,5,0,0,0,0,0,	35063002,
1·42138183	↓ 3,6,6,0,0,0,0,0,	35162954,
1·42280321	↓ 3,6,7,0,0,0,0,0,	35262904,
1·42422601	↓ 3,6,8,0,0,0,0,0,	35362854,
1·42565023	↓ 3,6,9,0,0,0,0,0,	35462804,
1·42701215	↓ 3,7,0,0,0,0,0,0,	35558285,
1·42843916	↓ 3,7,1,0,0,0,0,0,	35658235,
1·42986760	↓ 3,7,2,0,0,0,0,0,	35758185,
1·43129747	↓ 3,7,3,0,0,0,0,0,	35858135,
1·43272877	↓ 3,7,4,0,0,0,0,0,	35958085,
1·43416150	↓ 3,7,5,0,0,0,0,0,	36058035,
1·43559566	↓ 3,7,6,0,0,0,0,0,	36157987,
1·43703126	↓ 3,7,7,0,0,0,0,0,	36257937,
1·43846829	↓ 3,7,8,0,0,0,0,0,	36357887,
1·43990676	↓ 3,7,9,0,0,0,0,0,	36457837,
1·44128227	↓ 3,8,0,0,0,0,0,0,	36553318,
1·44272355	↓ 3,8,1,0,0,0,0,0,	36653268,
1·44416627	↓ 3,8,2,0,0,0,0,0,	36753218,
1·44561044	↓ 3,8,3,0,0,0,0,0,	36853168,
1·44705605	↓ 3,8,4,0,0,0,0,0,	36953118,
1·44850311	↓ 3,8,5,0,0,0,0,0,	37053068,
1·44995161	↓ 3,8,6,0,0,0,0,0,	37153020,
1·45140156	↓ 3,8,7,0,0,0,0,0,	37252970,
1·45285296	↓ 3,8,8,0,0,0,0,0,	37352920,
1·45430581	↓ 3,8,9,0,0,0,0,0,	37452870,
1·45569509	↓ 3,9,0,0,0,0,0,0,	37548351,
1·45715079	↓ 3,9,1,0,0,0,0,0,	37648301,
1·45860794	↓ 3,9,2,0,0,0,0,0,	37748251,
1·46006655	↓ 3,9,3,0,0,0,0,0,	37848201,
1·46152662	↓ 3,9,4,0,0,0,0,0,	37948151,
1·46298815	↓ 3,9,5,0,0,0,0,0,	38048101,
1·46445114	↓ 3,9,6,0,0,0,0,0,	38148053,
1·46591559	↓ 3,9,7,0,0,0,0,0,	38248003,
1·46738151	↓ 3,9,8,0,0,0,0,0,	38347953,
1·46884889	↓ 3,9,9,0,0,0,0,0,	38447903,
1·46410000	↓ 4,0,0,0,0,0,0,0,	38124072,

2 = 69314718 10 = 230258509
4 = 138629436 100 = 460517018
8 = 207944154 1000 = 690775527

N. Nos.	D. Nos.	D. Logs.
1·46556410	↓4,0,1,0,0,0,0,0,	38224022,
1·46702966	↓4,0,2,0,0,0,0,0,	38323972,
1·46849669	↓4,0,3,0,0,0,0,0,	38423922,
1·46996519	↓4,0,4,0,0,0,0,0,	38523872,
1·47143516	↓4,0,5,0,0,0,0,0,	38623822,
1·47290660	↓4,0,6,0,0,0,0,0,	38723772,
1·47437951	↓4,0,7,0,0,0,0,0,	38823724,
1·47585389	↓4,0,8,0,0,0,0,0,	38923674,
1·47732974	↓4,0,9,0,0,0,0,0,	39023624,
1·47874100	↓4,1,0,0,0,0,0,0,	39119105,
1·48021974	↓4,1,1,0,0,0,0,0,	39219055,
1·48169996	↓4,1,2,0,0,0,0,0,	39319005,
1·48318166	↓4,1,3,0,0,0,0,0,	39418955,
1·48466484	↓4,1,4,0,0,0,0,0,	39518905,
1·48614951	↓4,1,5,0,0,0,0,0,	39618855,
1·48763566	↓4,1,6,0,0,0,0,0,	39718805,
1·48912329	↓4,1,7,0,0,0,0,0,	39818757,
1·49061241	↓4,1,8,0,0,0,0,0,	39918707,
1·49210302	↓4,1,9,0,0,0,0,0,	40018657,
1·49352841	↓4,2,0,0,0,0,0,0,	40114138,
1·49502194	↓4,2,1,0,0,0,0,0,	40214088,
1·49651696	↓4,2,2,0,0,0,0,0,	40314038,
1·49801348	↓4,2,3,0,0,0,0,0,	40413988,
1·49951149	↓4,2,4,0,0,0,0,0,	40513938,
1·50101100	↓4,2,5,0,0,0,0,0,	40613888,
1·50251201	↓4,2,6,0,0,0,0,0,	40713840,
1·50401452	↓4,2,7,0,0,0,0,0,	40813790,
1·50551853	↓4,2,8,0,0,0,0,0,	40913740,
1·50702405	↓4,2,9,0,0,0,0,0,	41013690,
1·50846369	↓4,3,0,0,0,0,0,0,	41109171,
1·50997215	↓4,3,1,0,0,0,0,0,	41209121,
1·51148212	↓4,3,2,0,0,0,0,0,	41309071,
1·51299360	↓4,3,3,0,0,0,0,0,	41409021,
1·51450659	↓4,3,4,0,0,0,0,0,	41508971,
1·51602110	↓4,3,5,0,0,0,0,0,	41608921,
1·51753712	↓4,3,6,0,0,0,0,0,	41708873,
1·51905466	↓4,3,7,0,0,0,0,0,	41808823,
1·52057371	↓4,3,8,0,0,0,0,0,	41908773,
1·52209428	↓4,3,9,0,0,0,0,0,	42008723,
1·52354833	↓4,4,0,0,0,0,0,0,	42104204,
1·52507188	↓4,4,1,0,0,0,0,0,	42204154,
1·52659695	↓4,4,2,0,0,0,0,0,	42304104,
1·52812355	↓4,4,3,0,0,0,0,0,	42404054,
1·52965167	↓4,4,4,0,0,0,0,0,	42504004,
1·53118132	↓4,4,5,0,0,0,0,0,	42603954,
1·53271250	↓4,4,6,0,0,0,0,0,	42703906,
1·53424521	↓4,4,7,0,0,0,0,0,	42803856,
1·53577946	↓4,4,8,0,0,0,0,0,	42903806,
1·53731524	↓4,4,9,0,0,0,0,0,	43003756,
1·53878381	↓4,5,0,0,0,0,0,0,	43099237,

2 = 69314718	10 = 230258509
4 = 138629436	100 = 460517018
8 = 207944154	1000 = 690775527

N. Nos.	D. Nos.	D. Logs.
1·54032259	↓4,5,1,0,0,0,0,0,	43199187,
1·54186292	↓4,5,2,0,0,0,0,0,	43299137,
1·54340478	↓4,5,3,0,0,0,0,0,	43399087,
1·54494818	↓4,5,4,0,0,0,0,0,	43499037,
1·54649314	↓4,5,5,0,0,0,0,0,	43598987,
1·54803963	↓4,5,6,0,0,0,0,0,	43698939,
1·54958767	↓4,5,7,0,0,0,0,0,	43798889,
1·55113726	↓4,5,8,0,0,0,0,0,	43898839,
1·55268840	↓4,5,9,0,0,0,0,0,	43998789,
1·55417165	↓4,6,0,0,0,0,0,0,	44094270,
1·55572582	↓4,6,1,0,0,0,0,0,	44194220,
1·55728155	↓4,6,2,0,0,0,0,0,	44294170,
1·55883883	↓4,6,3,0,0,0,0,0,	44394120,
1·56039767	↓4,6,4,0,0,0,0,0,	44494070,
1·56195807	↓4,6,5,0,0,0,0,0,	44594020,
1·56352003	↓4,6,6,0,0,0,0,0,	44693970,
1·56508355	↓4,6,7,0,0,0,0,0,	44793922,
1·56664863	↓4,6,8,0,0,0,0,0,	44893872,
1·56821528	↓4,6,9,0,0,0,0,0,	44993822,
1·56971337	↓4,7,0,0,0,0,0,0,	45089303,
1·57128308	↓4,7,1,0,0,0,0,0,	45189253,
1·57285436	↓4,7,2,0,0,0,0,0,	45289203,
1·57442721	↓4,7,3,0,0,0,0,0,	45389153,
1·57600164	↓4,7,4,0,0,0,0,0,	45489103,
1·57757764	↓4,7,5,0,0,0,0,0,	45589053,
1·57915522	↓4,7,6,0,0,0,0,0,	45689003,
1·58073438	↓4,7,7,0,0,0,0,0,	45788953,
1·58231511	↓4,7,8,0,0,0,0,0,	45888903,
1·58389743	↓4,7,9,0,0,0,0,0,	45988853,
1·58541050	↓4,8,0,0,0,0,0,0,	46084336,
1·58699591	↓4,8,1,0,0,0,0,0,	46184286,
1·58858291	↓4,8,2,0,0,0,0,0,	46284236,
1·59017149	↓4,8,3,0,0,0,0,0,	46384186,
1·59176166	↓4,8,4,0,0,0,0,0,	46484136,
1·59335342	↓4,8,5,0,0,0,0,0,	46584086,
1·59494677	↓4,8,6,0,0,0,0,0,	46684026,
1·59654172	↓4,8,7,0,0,0,0,0,	46783986,
1·59813826	↓4,8,8,0,0,0,0,0,	46883936,
1·59973640	↓4,8,9,0,0,0,0,0,	46983886,
1·60126461	↓4,9,0,0,0,0,0,0,	47079369,
1·60286587	↓4,9,1,0,0,0,0,0,	47179319,
1·60446874	↓4,9,2,0,0,0,0,0,	47279269,
1·60607321	↓4,9,3,0,0,0,0,0,	47379219,
1·60767928	↓4,9,4,0,0,0,0,0,	47479169,
1·60928696	↓4,9,5,0,0,0,0,0,	47579119,
1·61089625	↓4,9,6,0,0,0,0,0,	47679069,
1·61250715	↓4,9,7,0,0,0,0,0,	47779019,
1·61411966	↓4,9,8,0,0,0,0,0,	47878969,
1·61573378	↓4,9,9,0,0,0,0,0,	47978919,
1·61051000	↓5,0,0,0,0,0,0,0,	47655090,

2 = 69314718 10 = 230258509
4 = 138629436 100 = 460517018
8 = 207944154 1000 = 690775527

N. Nos.	D. Nos.	D. Logs.
1·61212051	↓5,0,1,0,0,0,0,0,	47755040,
1·61373263	↓5,0,2,0,0,0,0,0,	47854990,
1·61534636	↓5,0,3,0,0,0,0,0,	47954940,
1·61696171	↓5,0,4,0,0,0,0,0,	48054890,
1·61857867	↓5,0,5,0,0,0,0,0,	48154840,
1·62019725	↓5,0,6,0,0,0,0,0,	48254790,
1·62181745	↓5,0,7,0,0,0,0,0,	48354740,
1·62343927	↓5,0,8,0,0,0,0,0,	48454690,
1·62506271	↓5,0,9,0,0,0,0,0,	48554640,
1·62661510	↓5,1,0,0,0,0,0,0,	48650123,
1·62824172	↓5,1,1,0,0,0,0,0,	48750073,
1·62986996	↓5,1,2,0,0,0,0,0,	48850023,
1·63149983	↓5,1,3,0,0,0,0,0,	48949973,
1·63313133	↓5,1,4,0,0,0,0,0,	49049923,
1·63476446	↓5,1,5,0,0,0,0,0,	49149873,
1·63639922	↓5,1,6,0,0,0,0,0,	49249823,
1·63803562	↓5,1,7,0,0,0,0,0,	49349773,
1·63967365	↓5,1,8,0,0,0,0,0,	49449723,
1·64131332	↓5,1,9,0,0,0,0,0,	49549673,
1·64288125	↓5,2,0,0,0,0,0,0,	49645156,
1·64452413	↓5,2,1,0,0,0,0,0,	49745106,
1·64616865	↓5,2,2,0,0,0,0,0,	49845056,
1·64781482	↓5,2,3,0,0,0,0,0,	49945006,
1·64946263	↓5,2,4,0,0,0,0,0,	50044956,
1·65111210	↓5,2,5,0,0,0,0,0,	50144906,
1·65276320	↓5,2,6,0,0,0,0,0,	50244856,
1·65441596	↓5,2,7,0,0,0,0,0,	50344806,
1·65607037	↓5,2,8,0,0,0,0,0,	50444756,
1·65772644	↓5,2,9,0,0,0,0,0,	50544706,
1·65931006	↓5,3,0,0,0,0,0,0,	50640189,
1·66096937	↓5,3,1,0,0,0,0,0,	50740139,
1·66263034	↓5,3,2,0,0,0,0,0,	50840089,
1·66429297	↓5,3,3,0,0,0,0,0,	50940039,
1·66595726	↓5,3,4,0,0,0,0,0,	51039989,
1·66762322	↓5,3,5,0,0,0,0,0,	51139939,
1·66929084	↓5,3,6,0,0,0,0,0,	51239889,
1·67096013	↓5,3,7,0,0,0,0,0,	51339839,
1·67263109	↓5,3,8,0,0,0,0,0,	51439789,
1·67430372	↓5,3,9,0,0,0,0,0,	51539739,
1·67590316	↓5,4,0,0,0,0,0,0,	51635222,
1·67757906	↓5,4,1,0,0,0,0,0,	51735172,
1·67925664	↓5,4,2,0,0,0,0,0,	51835122,
1·68093590	↓5,4,3,0,0,0,0,0,	51935072,
1·68261684	↓5,4,4,0,0,0,0,0,	52035022,
1·68429946	↓5,4,5,0,0,0,0,0,	52134972,
1·68598376	↓5,4,6,0,0,0,0,0,	52234922,
1·68766974	↓5,4,7,0,0,0,0,0,	52334872,
1·68935741	↓5,4,8,0,0,0,0,0,	52434822,
1·69104677	↓5,4,9,0,0,0,0,0,	52534772,
1·69266219	↓5,5,0,0,0,0,0,0,	52630255,

2 = 69314718 10 = 230258510
4 = 138629436 100 = 460517020
8 = 207944154 1000 = 690775530

N. Nos.	D. Nos.	D. Logs.
1·69435485	↓5,5,1,0,0,0,0,0,	52730205,
1·69604920	↓5,5,2,0,0,0,0,0,	52830155,
1·69774525	↓5,5,3,0,0,0,0,0,	52930105,
1·69944300	↓5,5,4,0,0,0,0,0,	53030055,
1·70114244	↓5,5,5,0,0,0,0,0,	53120005,
1·70284358	↓5,5,6,0,0,0,0,0,	53229955,
1·70454642	↓5,5,7,0,0,0,0,0,	53329905,
1·70625096	↓5,5,8,0,0,0,0,0,	53429855,
1·70795721	↓5,5,9,0,0,0,0,0,	53529805,
1·70958881	↓5,6,0,0,0,0,0,0,	53625288,
1·71129840	↓5,6,1,0,0,0,0,0,	53725238,
1·71300970	↓5,6,2,0,0,0,0,0,	53825188,
1·71472271	↓5,6,3,0,0,0,0,0,	53925138,
1·71643743	↓5,6,4,0,0,0,0,0,	54025088,
1·71815387	↓5,6,5,0,0,0,0,0,	54125038,
1·71987202	↓5,6,6,0,0,0,0,0,	54224988,
1·72159189	↓5,6,7,0,0,0,0,0,	54324938,
1·72331348	↓5,6,8,0,0,0,0,0,	54424888,
1·72503679	↓5,6,9,0,0,0,0,0,	54524838,
1·72668470	↓5,7,0,0,0,0,0,0,	54620321,
1·72841138	↓5,7,1,0,0,0,0,0,	54720271,
1·73013979	↓5,7,2,0,0,0,0,0,	54820221,
1·73186993	↓5,7,3,0,0,0,0,0,	54920171,
1·73360180	↓5,7,4,0,0,0,0,0,	55020121,
1·73533540	↓5,7,5,0,0,0,0,0,	55120071,
1·73707074	↓5,7,6,0,0,0,0,0,	55220021,
1·73880781	↓5,7,7,0,0,0,0,0,	55319971,
1·74054662	↓5,7,8,0,0,0,0,0,	55419921,
1·74228717	↓5,7,9,0,0,0,0,0,	55519871,
1·74395155	↓5,8,0,0,0,0,0,0,	55615354,
1·74569550	↓5,8,1,0,0,0,0,0,	55715304,
1·74744120	↓5,8,2,0,0,0,0,0,	55815254,
1·74918864	↓5,8,3,0,0,0,0,0,	55915204,
1·75093783	↓5,8,4,0,0,0,0,0,	56015154,
1·75268877	↓5,8,5,0,0,0,0,0,	56115104,
1·75444146	↓5,8,6,0,0,0,0,0,	56215054,
1·75619590	↓5,8,7,0,0,0,0,0,	56315004,
1·75795210	↓5,8,8,0,0,0,0,0,	56414954,
1·75971006	↓5,8,9,0,0,0,0,0,	56514904,
1·76139107	↓5,9,0,0,0,0,0,0,	56610387,
1·76315246	↓5,9,1,0,0,0,0,0,	56710337,
1·76491561	↓5,9,2,0,0,0,0,0,	56810287,
1·76668052	↓5,9,3,0,0,0,0,0,	56910237,
1·76844720	↓5,9,4,0,0,0,0,0,	57010187,
1·77021565	↓5,9,5,0,0,0,0,0,	57110137,
1·77198587	↓5,9,6,0,0,0,0,0,	57210087,
1·77375786	↓5,9,7,0,0,0,0,0,	57310037,
1·77553162	↓5,9,8,0,0,0,0,0,	57409987,
1·77730713	↓5,9,9,0,0,0,0,0,	57509937,
1·77156100	↓6,0,0,0,0,0,0,0,	57186108,

2= 69314718	10=230258510
4=138629436	100=460517020
8=207944154	1000=690775530

N. Nos.	D. Nos.	D. Logs.
1·77333256	↓6,0,1,0,0,0,0,0,	57286058,
1·77510589	↓6,0,2,0,0,0,0,0,	57386008,
1·77688100	↓6,0,3,0,0,0,0,0,	57485958,
1·77865788	↓6,0,4,0,0,0,0,0,	57585908,
1·78043654	↓6,0,5,0,0,0,0,0,	57685858,
1·78221698	↓6,0,6,0,0,0,0,0,	57785808,
1·78399920	↓6,0,7,0,0,0,0,0,	57885758,
1·78578320	↓6,0,8,0,0,0,0,0,	57985708,
1·78756898	↓6,0,9,0,0,0,0,0,	58085658,
1·78927661	↓6,1,0,0,0,0,0,0,	58181141,
1·79106589	↓6,1,1,0,0,0,0,0,	58281091,
1·79285696	↓6,1,2,0,0,0,0,0,	58381041,
1·79464982	↓6,1,3,0,0,0,0,0,	58480991,
1·79644447	↓6,1,4,0,0,0,0,0,	58580941,
1·79824091	↓6,1,5,0,0,0,0,0,	58680891,
1·80003915	↓6,1,6,0,0,0,0,0,	58780841,
1·80183919	↓6,1,7,0,0,0,0,0,	58880791,
1·80364103	↓6,1,8,0,0,0,0,0,	58980741,
1·80544467	↓6,1,9,0,0,0,0,0,	59080691,
1·80716938	↓6,2,0,0,0,0,0,0,	59176174,
1·80897655	↓6,2,1,0,0,0,0,0,	59276124,
1·81078553	↓6,2,2,0,0,0,0,0,	59376074,
1·81259632	↓6,2,3,0,0,0,0,0,	59476024,
1·81440892	↓6,2,4,0,0,0,0,0,	59575974,
1·81622333	↓6,2,5,0,0,0,0,0,	59675924,
1·81803955	↓6,2,6,0,0,0,0,0,	59775874,
1·81985759	↓6,2,7,0,0,0,0,0,	59875824,
1·82167745	↓6,2,8,0,0,0,0,0,	59975774,
1·82349913	↓6,2,9,0,0,0,0,0,	60075724,
1·82524107	↓6,3,0,0,0,0,0,0,	60171207,
1·82706631	↓6,3,1,0,0,0,0,0,	60271157,
1·82889338	↓6,3,2,0,0,0,0,0,	60371107,
1·83072227	↓6,3,3,0,0,0,0,0,	60471057,
1·83255299	↓6,3,4,0,0,0,0,0,	60571007,
1·83438554	↓6,3,5,0,0,0,0,0,	60670957,
1·83621993	↓6,3,6,0,0,0,0,0,	60770907,
1·83805615	↓6,3,7,0,0,0,0,0,	60870857,
1·83989421	↓6,3,8,0,0,0,0,0,	60970807,
1·84173410	↓6,3,9,0,0,0,0,0,	61070757,
1·84349348	↓6,4,0,0,0,0,0,0,	61166240,
1·84533697	↓6,4,1,0,0,0,0,0,	61266190,
1·84718231	↓6,4,2,0,0,0,0,0,	61366140,
1·84902949	↓6,4,3,0,0,0,0,0,	61466090,
1·85087852	↓6,4,4,0,0,0,0,0,	61566040,
1·85272940	↓6,4,5,0,0,0,0,0,	61665990,
1·85458213	↓6,4,6,0,0,0,0,0,	61765940,
1·85643671	↓6,4,7,0,0,0,0,0,	61865890,
1·85829315	↓6,4,8,0,0,0,0,0,	61965840,
1·86015144	↓6,4,9,0,0,0,0,0,	62065790,
1·86192841	↓6,5,0,0,0,0,0,0,	62161273,

2= 69314718 10=230258509
4=138629436 100=460517018
8=207944154 1000=690775527

N. Nos.	D. Nos.	D. Logs.
1·86379034	↓6,5,1,0,0,0,0,0,	62261223,
1·86565413	↓6,5,2,0,0,0,0,0,	62361173,
1·86751978	↓6,5,3,0,0,0,0,0,	62461123,
1·86938730	↓6,5,4,0,0,0,0,0,	62561073,
1·87125669	↓6,5,5,0,0,0,0,0,	62661023,
1·87312795	↓6,5,6,0,0,0,0,0,	62760973,
1·87500108	↓6,5,7,0,0,0,0,0,	62860923,
1·87687608	↓6,5,8,0,0,0,0,0,	62960873,
1·87875296	↓6,5,9,0,0,0,0,0,	63060823,
1·88054769	↓6,6,0,0,0,0,0,0,	63156306,
1·88242824	↓6,6,1,0,0,0,0,0,	63256256,
1·88431067	↓6,6,2,0,0,0,0,0,	63356206,
1·88619498	↓6,6,3,0,0,0,0,0,	63456156,
1·88808117	↓6,6,4,0,0,0,0,0,	63556106,
1·88996925	↓6,6,5,0,0,0,0,0,	63656056,
1·89185922	↓6,6,6,0,0,0,0,0,	63756006,
1·89375108	↓6,6,7,0,0,0,0,0,	63855956,
1·89564483	↓6,6,8,0,0,0,0,0,	63955906,
1·89754047	↓6,6,9,0,0,0,0,0,	64055856,
1·89935317	↓6,7,0,0,0,0,0,0,	64151339,
1·90125252	↓6,7,1,0,0,0,0,0,	64251289,
1·90315377	↓6,7,2,0,0,0,0,0,	64351239,
1·90505692	↓6,7,3,0,0,0,0,0,	64451189,
1·90696198	↓6,7,4,0,0,0,0,0,	64551139,
1·90886894	↓6,7,5,0,0,0,0,0,	64651089,
1·91077781	↓6,7,6,0,0,0,0,0,	64751039,
1·91268859	↓6,7,7,0,0,0,0,0,	64850989,
1·91460128	↓6,7,8,0,0,0,0,0,	64950939,
1·91651588	↓6,7,9,0,0,0,0,0,	65050889,
1·91834670	↓6,8,0,0,0,0,0,0,	65146372,
1·92026505	↓6,8,1,0,0,0,0,0,	65246322,
1·92218532	↓6,8,2,0,0,0,0,0,	65346272,
1·92410751	↓6,8,3,0,0,0,0,0,	65446222,
1·92603162	↓6,8,4,0,0,0,0,0,	65546172,
1·92795765	↓6,8,5,0,0,0,0,0,	65646122,
1·92988561	↓6,8,6,0,0,0,0,0,	65746072,
1·93181550	↓6,8,7,0,0,0,0,0,	65846022,
1·93374732	↓6,8,8,0,0,0,0,0,	65945972,
1·93568107	↓6,8,9,0,0,0,0,0,	66045922,
1·93753017	↓6,9,0,0,0,0,0,0,	66141405,
1·93946770	↓6,9,1,0,0,0,0,0,	66241355,
1·94140717	↓6,9,2,0,0,0,0,0,	66341305,
1·94334858	↓6,9,3,0,0,0,0,0,	66441255,
1·94529193	↓6,9,4,0,0,0,0,0,	66541205,
1·94723722	↓6,9,5,0,0,0,0,0,	66641155,
1·94918446	↓6,9,6,0,0,0,0,0,	66741105,
1·95113364	↓6,9,7,0,0,0,0,0,	66841055,
1·95308477	↓6,9,8,0,0,0,0,0,	66941005,
1·95503785	↓6,9,9,0,0,0,0,0,	67040955,
1·94871710	↓7,0,0,0,0,0,0,0,	66717126,

2= 69314718 10=230258509
4=138629436 100=460517018
8=207944154 1000=690775527

N. Nos.	D. Nos.	D. Logs.
1·95066582	↓ 7,0,1,0,0,0,0,0,	66817076,
1·95261649	↓ 7,0,2,0,0,0,0,0,	66917026,
1·95456911	↓ 7,0,3,0,0,0,0,0,	67016976,
1·95652368	↓ 7,0,4,0,0,0,0,0,	67116926,
1·95848020	↓ 7,0,5,0,0,0,0,0,	67216876,
1·96043868	↓ 7,0,6,0,0,0,0,0,	67316826,
1·96239912	↓ 7,0,7,0,0,0,0,0,	67416776,
1·96436152	↓ 7,0,8,0,0,0,0,0,	67516726,
1·96632588	↓ 7,0,9,0,0,0,0,0,	67616676,
1·96820427	↓ 7,1,0,0,0,0,0,0,	67712159,
1·97017247	↓ 7,1,1,0,0,0,0,0,	67812109,
1·97214264	↓ 7,1,2,0,0,0,0,0,	67912059,
1·97411478	↓ 7,1,3,0,0,0,0,0,	68012009,
1·97608889	↓ 7,1,4,0,0,0,0,0,	68111959,
1·97806498	↓ 7,1,5,0,0,0,0,0,	68211909,
1·98004304	↓ 7,1,6,0,0,0,0,0,	68311859,
1·98202308	↓ 7,1,7,0,0,0,0,0,	68411809,
1·98400510	↓ 7,1,8,0,0,0,0,0,	68511759,
1·98598911	↓ 7,1,9,0,0,0,0,0,	68611709,
1·98788631	↓ 7,2,0,0,0,0,0,0,	68707192,
1·98987420	↓ 7,2,1,0,0,0,0,0,	68807142,
1·99186407	↓ 7,2,2,0,0,0,0,0,	68907092,
1·99385593	↓ 7,2,3,0,0,0,0,0,	69007042,
1·99584978	↓ 7,2,4,0,0,0,0,0,	69106992,
1·99784563	↓ 7,2,5,0,0,0,0,0,	69206942,
1·99984348	↓ 7,2,6,0,0,0,0,0,	69306892,
2·00184332	↓ 7,2,7,0,0,0,0,0,	69406842,
2·00384516	↓ 7,2,8,0,0,0,0,0,	69506792,
2·00584901	↓ 7,2,9,0,0,0,0,0,	69606742,
2·00776517	↓ 7,3,0,0,0,0,0,0,	69702225,
2·00977294	↓ 7,3,1,0,0,0,0,0,	69802175,
2·01178271	↓ 7,3,2,0,0,0,0,0,	69902125,
2·01379449	↓ 7,3,3,0,0,0,0,0,	70002075,
2·01580828	↓ 7,3,4,0,0,0,0,0,	70102025,
2·01782409	↓ 7,3,5,0,0,0,0 0,	70201975,
2·01984191	↓ 7,3,6,0,0,0,0,0,	70301925,
2·02186175	↓ 7,3,7,0,0,0,0,0,	70401875,
2·02388361	↓ 7,3,8,0,0,0,0,0,	70501825,
2·02590749	↓ 7,3,9,0,0,0,0,0,	70601775,
2·02784282	↓ 7,4,0,0,0,0,0,0,	70697258,
2·02987066	↓ 7,4,1,0,0,0,0,0,	70797208,
2·03190053	↓ 7,4,2,0,0,0,0,0,	70897158,
2·03393243	↓ 7,4,3,0 0,0,0,0,	70997108,
2·03596636	↓ 7,4,4,0,0,0,0,0,	71097058,
2·03800233	↓ 7,4,5,0,0,0,0,0,	71197008,
2·04004033	↓ 7,4,6,0,0,0,0,0,	71296958,
2·04208037	↓ 7,4,7,0,0,0,0,0,	71396908,
2·04412245	↓ 7,4,8,0,0,0,0,0,	71496858,
2·04616657	↓ 7,4,9,0,0,0,0,0,	71596808,
2·04812125	↓ 7,5,0,0,0,0,0,0,	71692291.

2 = 69314718	10 = 230258509
4 = 138629436	100 = 460517018
8 = 207944154	1000 = 690775527

N. Nos.	D. Nos.	D. Logs.
2·05016937	↓7,5,1,0,0,0,0,0,	71792241,
2·05221953	↓7,5,2,0,0,0,0,0,	71892191,
2·05427175	↓7,5,3,0,0,0,0,0,	71992141,
2·05632602	↓7,5,4,0,0,0,0,0,	72092091,
2·05838234	↓7,5,5,0,0,0,0,0,	72192041,
2·06044072	↓7,5,6,0,0,0,0,0,	72291991,
2·06250116	↓7,5,7,0,0,0,0,0,	72391941,
2·06456368	↓7,5,8,0,0,0,0,0,	72491891,
2·06662824	↓7,5,9,0,0,0,0,0,	72591841,
2·06860246	↓7,6,0,0,0,0,0,0,	72687324,
2·07067106	↓7,6,1,0,0,0,0,0,	72787274,
2·07274173	↓7,6,2,0,0,0,0,0,	72887224,
2·07481447	↓7,6,3,0,0,0,0,0,	72987174,
2·07688928	↓7,6,4,0,0,0,0,0,	73087124,
2·07896617	↓7,6,5,0,0,0,0,0,	73187074,
2·08104514	↓7,6,6,0,0,0,0,0,	73287024,
2·08312619	↓7,6,7,0,0,0,0,0,	73386974,
2·08520932	↓7,6,8,0,0,0,0,0,	73486924,
2·08729453	↓7,6,9,0,0,0,0,0,	73586874,
2·08928848	↓7,7,0,0,0,0,0,0,	73682357,
2·09137777	↓7,7,1,0,0,0,0,0,	73782307,
2·09346914	↓7,7,2,0,0,0,0,0,	73882257,
2·09556261	↓7,7,3,0,0,0,0,0,	73982207,
2·09765817	↓7,7,4,0,0,0,0,0,	74082157,
2·09975583	↓7,7,5,0,0,0,0,0,	74182107,
2·10185559	↓7,7,6,0,0,0,0,0,	74282057,
2·10395745	↓7,7,7,0,0,0,0,0,	74382007,
2·10606141	↓7,7,8,0,0,0,0,0,	74481957,
2·10816747	↓7,7,9,0,0,0,0,0	74581907,
2·11018136	↓7,8,0,0,0,0,0,0,	74677390,
2·11229154	↓7,8,1,0,0,0,0,0,	74777340,
2·11440383	↓7,8,2,0,0,0,0,0,	74877290,
2·11651823	↓7,8,3,0,0,0,0,0,	74977240,
2·11863475	↓7,8,4,0,0,0,0,0,	75077190,
2·12075338	↓7,8,5,0,0,0,0,0,	75177140,
2·12287413	↓7,8,6,0,0,0,0,0,	75277090,
2·12499700	↓7,8,7,0,0,0,0,0,	75377040,
2·12712199	↓7,8,8,0,0,0,0,0,	75476990,
2·12924911	↓7,8,9,0,0,0,0,0,	75576940,
2·13128317	↓7,9,0,0,0,0,0,0,	75672423,
2·13341445	↓7,9,1,0,0,0,0,0,	75772373,
2·13554786	↓7,9,2,0,0,0,0,0,	75872323,
2·13768341	↓7,9,3,0,0,0,0,0,	75972273,
2·13982109	↓7,9,4,0,0,0,0,0,	76072223,
2·14196091	↓7,9,5,0,0,0,0,0,	76172173,
2·14410287	↓7,9,6,0,0,0,0,0,	76272123,
2·14624697	↓7,9,7,0,0,0,0,0,	76372073,
2·14839322	↓7,9,8,0,0,0,0,0,	76471923,
2·15054161	↓7,9,9,0,0,0,0,0,	76571873,
2·14358881	↓8,0,0,0,0,0,0,0,	76248144,

2 = 69314718   10 = 230258509
4 = 138629436   100 = 460517018
8 = 207944154   1000 = 690775527

N. Nos.	D. Nos.	D. Logs.
2·14573240	↓8,0,1,0,0,0,0,0,	76348094,
2·14787813	↓8,0,2,0,0,0,0,0,	76448044,
2·15002601	↓8,0,3,0,0,0,0,0,	76547994,
2·15217604	↓8,0,4,0,0,0,0,0,	76647944,
2·15432822	↓8,0,5,0,0,0,0,0,	76747894,
2·15648255	↓8,0,6,0,0,0,0,0,	76847844,
2·15863903	↓8,0,7,0,0,0,0,0,	76947794,
2·16079767	↓8,0,8,0,0,0,0,0,	77047744,
2·16295847	↓8,0,9,0,0,0,0,0,	77147694,
2·16502470	↓8,1,0,0,0,0,0,0,	77243177,
2·16718972	↓8,1,1,0,0,0,0,0,	77343127,
2·16935691	↓8,1,2,0,0,0,0,0,	77443077,
2·17152627	↓8,1,3,0,0,0,0,0,	77543027,
2·17369780	↓8,1,4,0,0,0,0,0,	77642977,
2·17587149	↓8,1,5,0,0,0,0,0,	77742927,
2·17804736	↓8,1,6,0,0,0,0,0,	77842877,
2·18022541	↓8,1,7,0,0,0,0,0,	77942827,
2·18240563	↓8,1,8,0,0,0,0,0,	78042777,
2·18458804	↓8,1,9,0,0,0,0,0,	78142727,
2·18667495	↓8,2,0,0,0,0,0,0,	78238210,
2·18886162	↓8,2,1,0,0,0,0,0,	78338160,
2·19105048	↓8,2,2,0,0,0,0,0,	78438110,
2·19324153	↓8,2,3,0,0,0,0,0,	78538060,
2·19543477	↓8,2,4,0,0,0,0,0,	78638010,
2·19763020	↓8,2,5,0,0,0,0,0,	78737960,
2·19982783	↓8,2,6,0,0,0,0,0,	78837910,
2·20202766	↓8,2,7,0,0,0,0,0,	78937860,
2·20422969	↓8,2,8,0,0,0,0,0,	79037810,
2·20643392	↓8,2,9,0,0,0,0,0,	79137760,
2·20854170	↓8,3,0,0,0,0,0,0,	79233243,
2·21075024	↓8,3,1,0,0,0,0,0,	79333193,
2·21296099	↓8,3,2,0,0,0,0,0,	79433143,
2·21517395	↓8,3,3,0,0,0,0,0,	79533093,
2·21738912	↓8,3,4,0,0,0,0,0,	79633043,
2·21960651	↓8,3,5,0,0,0,0,0,	79732993,
2·22182612	↓8,3,6,0,0,0,0,0,	79832943,
2·22404795	↓8,3,7,0,0,0,0,0,	79932893,
2·22627200	↓8,3,8,0,0,0,0,0,	80032843,
2·22849827	↓8,3,9,0,0,0,0,0,	80132793,
2·23062712	↓8,4,0,0,0,0,0,0,	80228276,
2·23285775	↓8,4,1,0,0,0,0,0,	80328226,
2·23509061	↓8,4,2,0,0,0,0,0,	80428176,
2·23732570	↓8,4,3,0,0,0,0,0,	80528126,
2·23956303	↓8,4,4,0,0,0,0,0,	80628076,
2·24180259	↓8,4,5,0,0,0,0,0,	80728026,
2·24404439	↓8,4,6,0,0,0,0,0,	80827976,
2·24628843	↓8,4,7,0,0,0,0,0,	80927926,
2·24853472	↓8,4,8,0,0,0,0,0,	81027876,
2·25078325	↓8,4,9,0,0,0,0,0,	81127826,
2·25293339	↓8,5,0,0,0,0,0,0,	81223309,

2= 69314718 10=230258509

4=138629436 100=460517018

=207944154 1000=690775527

N. Nos.	D. Nos.	D. Logs.
2·25518632	↓8,5,1,0,0,0,0,0,	81323259,
2·25744151	↓8,5,2,0,0,0,0,0,	81423209,
2·25969895	↓8,5,3,0,0,0,0,0,	81523159,
2·26195865	↓8,5,4,0,0,0,0,0,	81623109,
2·26422061	↓8,5,5,0,0,0,0,0,	81723059,
2·26648483	↓8,5,6,0,0,0,0,0,	81823009,
2·26875131	↓8,5,7,0,0,0,0,0,	81922959,
2·27102006	↓8,5,8,0,0,0,0,0,	82022909,
2·27329108	↓8,5,9,0,0,0,0,0,	82122859,
2·27546272	↓8,6,0,0,0,0,0,0,	82218342,
2·27773818	↓8,6,1,0,0,0,0,0,	82318292,
2·28001591	↓8,6,2,0,0,0,0,0,	82418242,
2·28229592	↓8,6,3,0,0,0,0,0,	82518192,
2·28457822	↓8,6,4,0,0,0,0,0,	82618142,
2·28686280	↓8,6,5,0,0,0,0,0,	82718092,
2·28914966	↓8,6,6,0,0,0,0,0,	82818042,
2·29143881	↓8,6,7,0,0,0,0,0,	82917992,
2·29373025	↓8,6,8,0,0,0,0,0,	83017942,
2·29602398	↓8,6,9,0,0,0,0,0,	83117892,
2·29821735	↓8,7,0,0,0,0,0,0,	83203375,
2·30051556	↓8,7,1,0,0,0,0,0,	83303325,
2·30281608	↓8,7,2,0,0,0,0,0,	83403275,
2·30511890	↓8,7,3,0,0,0,0,0,	83503225,
2·30742402	↓8,7,4,0,0,0,0,0,	83603175,
2·30973144	↓8,7,5,0,0,0,0,0,	83703125,
2·31204117	↓8,7,6,0,0,0,0,0,	83803075,
2·31435321	↓8,7,7,0,0,0,0,0,	83903025,
2·31666756	↓8,7,8,0,0,0,0,0,	84002975,
2·31898423	↓8,7,9,0,0,0,0,0,	84102925,
2·32119952	↓8,8,0,0,0,0,0,0,	84198408,
2·32352072	↓8,8,1,0,0,0,0,0,	84298358,
2·32584424	↓8,8,2,0,0,0,0,0,	84398308,
2·32817008	↓8,8,3,0,0,0,0,0,	84498258,
2·33049825	↓8,8,4,0,0,0,0,0,	84598208,
2·33282875	↓8,8,5,0,0,0,0,0,	84698158,
2·33516158	↓8,8,6,0,0,0,0,0,	84798108,
2·33749674	↓8,8,7,0,0,0,0,0,	84898058,
2·33983424	↓8,8,8,0,0,0,0,0,	84998008,
2·34217407	↓8,8,9,0,0,0,0,0,	85097958,
2·34441151	↓8,9,0,0,0,0,0,0,	85193441,
2·34675592	↓8,9,1,0,0,0,0,0,	85293391,
2·34910268	↓8,9,2,0,0,0,0,0,	85393341,
2·35145178	↓8,9,3,0,0,0,0,0,	85493291,
2·35380323	↓8,9,4,0,0,0,0,0,	85593241,
2·35615703	↓8,9,5,0,0,0,0,0,	85693191,
2·35851319	↓8,9,6,0,0,0,0,0,	85793141,
2·36087170	↓8,9,7,0,0,0,0,0,	85893091,
2·36323257	↓8,9,8,0,0,0,0,0,	85993041,
2·36559580	↓8,9,9,0,0,0,0,0,	86092991,
2·35794769	↓9,0,0,0,0,0,0,0,	85779162,

2= 69314718	10=230258509
4=138629436	100=460517018
8=207944154	1000=690775527

N. Nos.	D. Nos.	D. Logs.
2·36030564	↓9,0,1,0,0,0,0,0,	85879112,
2·36266595	↓9,0,2,0,0,0,0,0,	85979062,
2·36502862	↓9,0,3,0,0,0,0,0,	86079012,
2·36739365	↓9,0,4,0,0,0,0,0,	86178962,
2·36976104	↓9,0,5,0,0,0,0,0,	86278912,
2·37213080	↓9,0,6,0,0,0,0,0,	86378862,
2·37450293	↓9,0,7,0,0,0,0,0,	86478812,
2·37687743	↓9,0,8,0,0,0,0,0,	86578762,
2·37725431	↓9,0,9,0,0,0,0,0,	86678712,
2·38152717	↓9,1,0,0,0,0,0,0,	86774195,
2·38390870	↓9,1,1,0,0,0,0,0,	86874145,
2·38629261	↓9,1,2,0,0,0,0,0,	86974095,
2·38867890	↓9,1,3,0,0,0,0,0,	87074045,
2·39106758	↓9,1,4,0,0,0,0,0,	87173995,
2·39345865	↓9,1,5,0,0,0,0,0,	87273945,
2·39585211	↓9,1,6,0,0,0,0,0,	87373895,
2·39824797	↓9,1,7,0,0,0,0,0,	87473845,
2·40064622	↓9,1,8,0,0,0,0,0,	87573795,
2·40304687	↓9,1,9,0,0,0,0,0,	87673745,
2·40534244	↓9,2,0,0,0,0,0,0,	87769228,
2·40774778	↓9,2,1,0,0,0,0,0,	87869178,
2·41015553	↓9,2,2,0,0,0,0,0,	87969128,
2·41256569	↓9,2,3,0,0,0,0,0,	88069078,
2·41497826	↓9,2,4,0,0,0,0,0,	88169028,
2·41739324	↓9,2,5,0,0,0,0,0,	88268978,
2·41981063	↓9,2,6,0,0,0,0,0,	88368928,
2·42223044	↓9,2,7,0,0,0,0,0,	88468878,
2·42465267	↓9,2,8,0,0,0,0,0,	88568828,
2·42707732	↓9,2,9,0,0,0,0,0,	88668778,
2·42939586	↓9,3,0,0,0,0,0,0,	88764261,
2·43182526	↓9,3,1,0,0,0,0,0,	88864211,
2·43425709	↓9,3,2,0,0,0,0,0,	88964161,
2·43669135	↓9,3,3,0,0,0,0,0,	89064111,
2·43912804	↓9,3,4,0,0,0,0,0,	89164061,
2·44156716	↓9,3,5,0,0,0,0,0,	89264011,
2·44400873	↓9,3,6,0,0,0,0,0,	89363961,
2·44645274	↓9,3,7,0,0,0,0,0,	89463911,
2·44889919	↓9,3,8,0,0,0,0,0,	89563861,
2·45134809	↓9,3,9,0,0,0,0,0,	89663811,
2·45368982	↓9,4,0,0,0,0,0,0,	89759294,
2·45614351	↓9,4,1,0,0,0,0,0,	89859244,
2·45859965	↓9,4,2,0,0,0,0,0,	89959194,
2·46105825	↓9,4,3,0,0,0,0,0,	90059144,
2·46351931	↓9,4,4,0,0,0,0,0,	90159094,
2·46598283	↓9,4,5,0,0,0,0,0,	90259044,
2·46844881	↓9,4,6,0,0,0,0,0,	90358994,
2·47091726	↓9,4,7,0,0,0,0,0,	90458944,
2·47338818	↓9,4,8,0,0,0,0,0,	90558894,
2·47586157	↓9,4,9,0,0,0,0,0,	90658844,
2·47822672	↓9,5,0,0,0,0,0,0,	90754327,

2= 69314718 10=230258509
4=138629436 100=460517018
8=207944154 1000=690775527

N. Nos.	D. Nos.	D. Logs.
2·48070495	↓ 9,5,1,0,0,0,0,0,	90854277,
2·48318565	↓ 9,5,2,0,0,0,0,0,	90954227,
2·48566884	↓ 9,5,3,0,0,0,0,0,	91054177,
2·48815451	↓ 9,5,4,0,0,0,0,0,	91154127,
2·49064266	↓ 9,5,5,0,0,0,0,0,	91254067,
2·49313330	↓ 9,5,6,0,0,0,0,0,	91354017,
2·49562643	↓ 9,5,7,0,0,0,0,0,	91453967,
2·49812206	↓ 9,5,8,0,0,0,0,0,	91553917,
2·50062018	↓ 9,5,9,0,0,0,0,0,	91653867,
2·50300898	↓ 9,6,0,0,0,0,0,0,	91749360,
2·50551199	↓ 9,6,1,0,0,0,0,0,	91849310,
2·50801750	↓ 9,6,2,0,0,0,0,0,	91949260,
2·51052552	↓ 9,6,3,0,0,0,0,0,	92049210,
2·51303605	↓ 9,6,4,0,0,0,0,0,	92149160,
2·51554909	↓ 9,6,5,0,0,0,0,0,	92249110,
2·51806454	↓ 9,6,6,0,0,0,0,0,	92349060,
2·52058270	↓ 9,6,7,0,0,0,0,0,	92449010,
2·52310328	↓ 9,6,8,0,0,0,0,0,	92548960,
2·52562638	↓ 9,6,9,0,0,0,0,0,	92648910,
2·52803907	↓ 9,7,0,0,0,0,0,0,	92744393,
2·53056711	↓ 9,7,1,0,0,0,0,0,	92844343,
2·53309768	↓ 9,7,2,0,0,0,0,0,	92944293,
2·53563078	↓ 9,7,3,0,0,0,0,0,	93044243,
2·53816641	↓ 9,7,4,0,0,0,0,0,	93144193,
2·54070458	↓ 9,7,5,0,0,0,0,0,	93244143,
2·54324528	↓ 9,7,6,0,0,0,0,0,	93344093,
2·54578853	↓ 9,7,7,0,0,0,0,0,	93444043,
2·54833432	↓ 9,7,8,0,0,0,0,0,	93543993,
2·55088265	↓ 9,7,9,0,0,0,0,0,	93643943,
2·55331946	↓ 9,8,0,0,0,0,0,0,	93739426,
2·55587278	↓ 9,8,1,0,0,0,0,0,	93839376,
2·55842865	↓ 9,8,2,0,0,0,0,0,	93939326,
2·56098708	↓ 9,8,3,0,0,0,0,0,	94039276,
2·56354807	↓ 9,8,4,0,0,0,0,0,	94139226,
2·56611162	↓ 9,8,5,0,0,0,0,0,	94239176,
2·56867773	↓ 9,8,6,0,0,0,0,0,	94339126,
2·57124641	↓ 9,8,7,0,0,0,0,0,	94439076,
2·57381765	↓ 9,8,8,0,0,0,0,0,	94539026,
2·57639147	↓ 9,8,9,0,0,0,0,0,	94638976,
2·57885265	↓ 9,9,0,0,0,0,0,0,	94734459,
2·58143150	↓ 9,9,1,0,0,0,0,0,	94834409,
2·58401293	↓ 9,9,2,0,0,0,0,0,	94934359,
2·58659604	↓ 9,9,3,0,0,0,0,0,	95034309,
2·58918354	↓ 9,9,4,0,0,0,0,0,	95134259,
2·59177272	↓ 9,9,5,0,0,0,0,0,	95234209,
2·59436449	↓ 9,9,6,0,0,0,0,0,	95334159,
2·59695885	↓ 9,9,7,0,0,0,0,0,	95434109,
2·59955581	↓ 9,9,8,0,0,0,0,0,	95534059,
2·60215537	↓ 9,9,9,0,0,0,0,0,	95634009,
2·59374246	↓ 10,0,0,0,0,0,0,0,	95310180,

2 = 69314718   10 = 230258509
4 = 138629436   100 = 460517018
8 = 207944154   1000 = 690775527

N. Nos.	D. Nos.	D. Logs.
2·59633620	↓ 10,0,1,0,0,0,0,0,	95410130,
2·59893254	↓ 10,0,2,0,0,0,0,0,	95510080,
2·60153147	↓ 10,0,3,0,0,0,0,0,	95610030,
2·60413200	↓ 10,0,4,0,0,0,0,0,	95709980,
2·60673713	↓ 10,0,5,0,0,0,0,0,	95809930,
2·60934387	↓ 10,0,6,0,0,0,0,0,	95909880,
2·61195321	↓ 10,0,7,0,0,0,0,0,	96009830,
2·61456516	↓ 10,0,8,0,0,0,0,0,	96109780,
2·61717972	↓ 10,0,9,0,0,0,0,0,	96209730,
2·61967988	↓ 10, 1,0,0,0,0,0,0,	96305213,
2·62229956	↓ 10, 1,1,0,0,0,0,0,	96405163,
2·62492186	↓ 10, 1,2,0,0,0,0,0,	96505113,
2·62754678	↓ 10, 1,3,0,0,0,0,0,	96605063,
2·63017433	↓ 10, 1,4,0,0,0,0,0,	96705013,
2·63280450	↓ 10, 1,5,0,0,0,0,0,	96804963,
2·63543730	↓ 10, 1,6,0,0,0,0,0,	96904913,
2·63807274	↓ 10, 1,7,0,0,0,0,0,	97004863,
2·64071081	↓ 10, 1,8,0,0,0,0,0,	97104813,
2·64335152	↓ 10, 1,9,0,0,0,0,0,	97204763,
2·64587668	↓ 10,2,0,0,0,0,0,0,	97300246,
2·64852256	↓ 10,2,1,0,0,0,0,0,	97400196,
2·65117108	↓ 10,2,2,0,0,0,0,0,	97500146,
2·65382225	↓ 10,2,3,0,0,0,0,0,	97600096,
2·65647607	↓ 10,2,4,0,0,0,0,0,	97700046,
2·65913255	↓ 10,2,5,0,0,0,0,0,	97799996,
2·66179168	↓ 10,2,6,0,0,0,0,0,	97899946,
2·66445347	↓ 10,2,7,0,0,0,0,0,	98999896,
2·66711792	↓ 10,2,8,0,0,0,0,0,	98099846,
2·66978504	↓ 10,2,9,0,0,0,0,0,	98149796,
2·67233545	↓ 10,3,0,0,0,0,0,0,	98295279,
2·67500778	↓ 10,3 1,0,0,0,0,0,	98395229,
2·67768279	↓ 10,3,2,0,0,0,0,0,	98495179,
2·68036047	↓ 10,3,3,0,0,0,0,0,	98595129,
2·68304083	↓ 10,3,4,0,0,0,0,0,	98695079,
2·68572387	↓ 10,3,5,0,0,0,0 0,	98795029,
2·68840959	↓ 10,3,6,0,0,0,0,0,	98894979,
2·69109800	↓ 10,3,7,0,0,0,0,0,	98994929,
2·69378910	↓ 10,3,8,0,0,0,0,0,	99094879,
2·69648289	↓ 10,3 9,0,0,0,0,0,	99194829,
2·69905880	↓ 10,4,0,0,0,0,0,0,	99290312,
2·70175785	↓ 10,4,1,0,0,0,0,0,	99390262,
2·70445961	↓ 10,4 2,0,0,0,0,0,	99490212,
2·70716407	↓ 10,4,3,0,0,0,0,0,	99590162,
2·70987123	↓ 10,4,4,0,0,0,0,0,	99690112,
2·71258110	↓ 10,4,5,0,0,0,0,0,	99790062,
2·71529368	↓ 10,4,6,0,0,0,0,0,	99890012,
2·71800897	↓ 10,4,7,0,0,0,0,0,	99989962,
2·72072698	↓ 10,4,8,0,0,0,0,0,	100089912,
2·72344771	↓ 10,4,9,0,0,0 0,0,	100189862,
2·72604939	↓ 10,5,0,0,0,0,0,0,	100285345,

2= 69314718	10=230258509
4=138629436	100=460517018
8=207944154	1000=690775527

N. Nos.	D. Nos.	D. Logs.
2·72877544	↓10,5,1,0,0,0,0,0,	100385295,
2·73150422	↓10,5,2,0,0,0,0,0,	100485245,
2·73423572	↓10,5,3,0,0,0,0,0,	100585195,
2·73696996	↓10,5,4,0,0,0,0,0,	100685145,
2·73970693	↓10,5,5,0,0,0,0,0,	100785095,
2·74244664	↓10,5,6,0,0,0,0,0,	100885045,
2·74518909	↓10,5,7,0,0,0,0,0,	100984995,
2·74793428	↓10,5,8,0,0,0,0,0,	101084945,
2·75068221	↓10,5,9,0,0,0,0,0,	101184895,
2·75330988	↓10,6,0,0,0,0,0,0,	101280378,
2·75606318	↓10,6,1,0,0,0,0,0,	101380328,
2·75881924	↓10,6,2,0,0,0,0,0,	101480278,
2·76157806	↓10,6,3,0,0,0,0,0,	101580228,
2·76433964	↓10,6,4,0,0,0,0,0,	101680178,
2·76710398	↓10,6,5,0,0,0,0,0,	101780128,
2·76987108	↓10,6,6,0,0,0,0,0,	101880078,
2·77264095	↓10,6,7,0,0,0,0,0,	101980028,
2·77541359	↓10,6,8,0,0,0,0,0,	102079978,
2·77818900	↓10,6,9,0,0,0,0,0,	102179928,
2·78084298	↓10,7,0,0,0,0,0,0,	102275411,
2·78362382	↓10,7,1,0,0,0,0,0,	102375361,
2·78640744	↓10,7,2,0,0,0,0,0,	102475311,
2·78919385	↓10,7,3,0,0,0,0,0,	102575261,
2·79198304	↓10,7,4,0,0,0,0,0,	102675211,
2·79477502	↓10,7,5,0,0,0,0,0,	102775161,
2·79756980	↓10,7,6,0,0,0,0,0,	102875111,
2·80036736	↓10,7,7,0,0,0,0,0,	102975061,
2·80316773	↓10,7,8,0,0,0,0,0,	103075011,
2·80597090	↓10,7,9,0,0,0,0,0,	103174961,
2·80865141	↓10,8,0,0,0,0,0,0,	103270444,
2·81146006	↓10,8,1,0,0,0,0,0,	103370394,
2·81427152	↓10,8,2,0,0,0,0,0,	103470344,
2·81708579	↓10,8,3,0,0,0,0,0,	103570294,
2·81990287	↓10,8,4,0,0,0,0,0,	103670244,
2·82272278	↓10,8,5,0,0,0,0,0,	103770194,
2·82554549	↓10,8,6,0,0,0,0,0,	103870144,
2·82837103	↓10,8,7,0,0,0,0,0,	103970094,
2·83119940	↓10,8,8,0,0,0,0,0,	104070044,
2·83403060	↓10,8,9,0,0,0,0,0,	104169994,
2·83673792	↓10,9,0,0,0,0,0,0,	104265477,
2·83957466	↓10,9,1,0,0,0,0,0,	104365427,
2·84241423	↓10,9,2,0,0,0,0,0,	104465377,
2·84525664	↓10,9,3,0,0,0,0,0,	104565327,
2·84810190	↓10,9,4,0,0,0,0,0,	104665277,
2·85095000	↓10,9,5,0,0,0,0,0,	104765227,
2·85380095	↓10,9,6,0,0,0,0,0,	104865177,
2·85665475	↓10,9,7,0,0,0,0,0,	104965127,
2·85951140	↓10,9,8,0,0,0,0,0,	105065077,
2·86237091	↓10,9,9,0,0,0,0,0,	105165027,
2·85311671	↓ 1,0,0,0,0,0,0,0,0,	104841198,

2= 69314718	10=230258509
4=138629436	100=460517018
8=207944154	1000=690775527

N. Nos.	D. Nos.	D. Logs.
2·85596982	↓ 11,0,1,0,0,0,0,0,0,	104941148,
2·85882580	↓ 11,0,2,0,0,0,0,0,0,	105041098,
2·86168463	↓ 11,0,3,0,0,0,0,0,0,	105141048,
2·86454631	↓ 11,0,4,0,0,0,0,0,0,	•105240998,
2·86741086	↓ 11,0,5,0,0,0,0,0,0,	105340948,
2·87027827	↓ 11,0,6,0,0,0,0,0,0,	105440898,
2·87314855	↓ 11,0,7,0,0,0,0,0,0,	105540848,
2·87602170	↓ 11,0,8,0,0,0,0,0,0,	105640798,
2·87889772	↓ 11,0,9,0,0,0,0,0,0,	105740748,
2·88164788	↓ 11,1,0,0,0,0,0,0,0,	105836231,
2·88452952	↓ 11,1,1,0,0,0,0,0,0,	105936181,
2·88741405	↓ 11,1,2,0,0,0,0,0,0,	106036131,
2·89030146	↓ 11,1,3,0,0,0,0,0,0,	106136081,
2·89319176	↓ 11,1,4,0,0,0,0,0,0,	106236031,
2·89608495	↓ 11,1,5,0,0,0,0,0,0,	106335981,
2·89898103	↓ 11,1,6,0,0,0,0,0,0,	106435931,
2·90188001	↓ 11,1,7,0,0,0,0,0,0,	106535881,
2·90478189	↓ 11,1,8,0,0,0,0,0,0,	106635831,
2·90768667	↓ 11,1,9,0,0,0,0,0,0,	106735781,
2·91046436	↓ 11,2,0,0,0,0,0,0,0,	106831264,
2·91337482	↓ 11,2,1,0,0,0,0,0,0,	106931214,
2·91628817	↓ 11,2,2,0,0,0,0,0,0,	107031164,
2·91920448	↓ 11,2,3,0,0,0,0,0,0,	107131114,
2·92212368	↓ 11,2,4,0,0,0,0,0,0,	107231064,
2·92504580	↓ 11,2,5,0,0,0,0,0,0,	107331014,
2·92797084	↓ 11,2,6,0,0,0,0,0,0,	107430964,
2·93089881	↓ 11,2,7,0,0,0,0,0,0,	107530914,
2·93382971	↓ 11,2,8,0,0,0,0,0,0,	107630864,
2 93676354	↓ 11,2,9,0,0,0,0,0,0,	107730814,
2·93956900	↓ 11,3,0,0,0,0,0,0,0,	107826297,
2·94250857	↓ 11,3,1,0,0,0,0,0,0,	107926247,
2·94545109	↓ 11,3,2,0,0,0,0,0,0,	108026197,
2·94839653	↓ 11,3,3,0,0,0,0,0,0,	108126147,
2·95134493	↓ 11,3,4,0,0,0,0,0,0,	108226097,
2·95429627	↓ 11,3,5,0,0,0,0,0,0,	108326047,
2·95725057	↓ 11,3,6,0,0,0,0,0,0,	108425997,
2·96020782	↓ 11,3,7,0,0,0,0,0,0,	108525847,
2·96316803	↓ 11,3,8,0,0,0,0,0,0,	108625797,
2·96613120	↓ 11,3,9,0,0,0,0,0,0,	108725747,
2·96896469	↓ 11,4,0,0,0,0,0,0,0,	108821330,
2·97193365	↓ 11,4,1,0,0,0,0,0,0,	108921280,
2·97490558	↓ 11,4,2,0,0,0,0,0,0,	109021230,
2·97788049	↓ 11,4,3,0,0,0,0,0,0,	109121180,
2·98085837	↓ 11,4,4,0,0,0,0,0,0,	109221130,
2·98383926	↓ 11,4,5,0,0,0,0,0,0,	109321080,
2·98682307	↓ 11,4,6,0,0,0,0,0,0,	109421030,
2·98980989	↓ 11,4,7,0,0,0,0,0,0,	109520980,
2·99161132	↓ 11,4,7,6,0,2,3,9,	109581215,

2 = 69314718 10 = 230258509
4 = 138629436 100 = 460517018
8 = 207944154 1000 = 690775527

DESCENDING BRANCH.

$1 / \overline{1} / 5 / \overline{5} / 0 / \overline{2} / 2 / \overline{0} / 5 / \overline{5} / 1 / \overline{1}$

$/ \overline{1} / 5 / \overline{6} / 3 / \overline{6} / 6 / \overline{3} / 6 / \overline{5} / 1 /$ $\quad '11\uparrow$

$/ \overline{1} / 3 / \overline{4} / 4 / \overline{3} / 1 /$

·3	1	3	8	0	1	5	9	6	0	9+	
	1	2	5	5	2	4	2	3	8	4−	
			1	8	8	2	8	6	3	6+	$'4_2\uparrow$
					1	2	5	5	2	4−	
								3	1	4+	

·3	0	1	4	4	5	2	0	6	5	1+	
		2	1	1	0	1	1	6	4	5−	
					6	3	3	0	3	5+	$'7_3\uparrow$
							1	0	5	5−	
										1+	

·2	9	9	3	4	1	4	0	9	8	7+	
			1	7	9	6	0	4	8	5−	$'6_4\uparrow$
							4	4	9	0+	
										1−	

·2	9	9	1	6	1	8	4	9	9	1+	
·	·	·	·	†		5	9	8	3	2−	$'2_6\uparrow$
							8	9	7	5−	$'3_7\uparrow$
							2	3	9	3−	$'8_8\uparrow$
								1	5	0−	$'5_9\uparrow$
									2	1−	$'7_{10}\uparrow$
										1	

·2 9 9 1 6 1 1 3 6 1 9

∴ ||

$'11 \quad '4 \quad '7 \quad '6 \quad '0 \quad '2 \quad '3 \quad '8 \quad '5 \quad '7 \quad '0 \quad '5\uparrow$

Ascending Branch.

1	1	5	5	0	2	2	0	5	5	1	1	
1	5	6	3	6	6	3	6	5	1			↓11,
	1	3	4	4	3	1						
2	8	5	3	1	1	6	7	0	6	1	1	
	1	1	4	1	2	4	6	6	8	2	4	
			1	7	1	1	8	7	0	0	2	$\downarrow^2 4$,
					1	1	4	1	2	4	7	
								2	8	5	3	
2	9	6	8	9	6	4	6	8	5	3	7	
		2	0	7	8	2	7	5	2	8	0	
					6	2	3	4	8	2	6	$\downarrow^3 7$,
							1	0	3	9	1	
										1	0	
2	9	8	9	8	0	9	8	9	0	4	4	
			1	7	9	3	8	8	5	9	3	
							4	4	8	4	7	$\downarrow^4 6$,
											6	
2	9	9	1	6	0	4	2	2	4	9	0	
	•	•	•	†		5	9	8	3	2	1	$\downarrow^6 2$,
							8	9	7	4	8	$\downarrow^7 3$,
							2	3	9	3	3	$\downarrow^8 8$,
								1	4	9	6	$\downarrow^9 5$,
									2	0	9	$\downarrow^{10} 7$,
2 · 9		9	1	6	1	1	3	6	1	9	7	

∴ ‖

↓11, 4, 7, 6, 0, 2, 3, 8, 5, 7, 0, 5,

Hence the two branches coïncide, the dual digits and the digits of the corresponding natural numbers at this point being expressed by the same numbers in consecutive order.

TABLE II.

TABLE II.

DESCENDING BRANCH

OF

DUAL ARITHMETIC.

DUAL NUMBERS AND DUAL LOGARITHMS,

WITH

CORRESPONDING NATURAL NUMBERS.

N. Nos.	D. Nos.	D. Logs.
·99900000	·0·0·1·0·0·0·0·0 ↑	·100050
·99800100	·0·0·2·0·0·0·0·0 ↑	·200100
·99700300	·0·0·3·0·0·0·0·0 ↑	·300150
·99600600	·0·0·4·0·0·0·0·0 ↑	·400200
·99500999	·0·0·5·0·0·0·0·0 ↑	·500250
·99401498	·0·0·6·0·0·0·0·0 ↑	·600300
·99302097	·0·0·7·0·0·0·0·0 ↑	·700350
·99202795	·0·0·8·0·0·0·0·0 ↑	·800400
·99103592	·0·0·9·0·0·0·0·0 ↑	·900450
·99000000	·0·1·0·0·0·0·0·0 ↑	·1005034
·98901000	·0·1·1·0·0·0·0·0 ↑	·1105084
·98802099	·0·1·2·0·0·0·0·0 ↑	·1205134
·98703297	·0·1·3·0·0·0·0·0 ↑	·1305180
·98604594	·0·1·4·0·0·0·0·0 ↑	·1405230
·98505989	·0·1·5·0·0·0·0·0 ↑	·1505280
·98407483	·0·1·6·0·0·0·0·0 ↑	·1605330
·98309076	·0·1·7·0·0·0·0·0 ↑	·1705380
·98210767	·0·1·8·0·0·0·0·0 ↑	·1805430
·98112556	·0·1·9·0·0·0·0·0 ↑	·1905480
·98010000	·0·2·0·0·0·0·0·0 ↑	·2010068
·97911990	·0·2·1·0·0·0·0·0 ↑	·2110118
·97814078	·0·2·2·0·0·0·0·0 ↑	·2210168
·97716264	·0·2·3·0·0·0·0·0 ↑	·2310218
·97618548	·0·2·4·0·0·0·0·0 ↑	·2410268
·97520929	·0·2·5·0·0·0·0·0 ↑	·2510318
·97423408	·0·2·6·0·0·0·0·0 ↑	·2610368
·97325985	·0·2·7·0·0·0·0·0 ↑	·2710418
·97228659	·0·2·8·0·0·0·0·0 ↑	·2810468
·97131430	·0·2·9·0·0·0·0·0 ↑	·2910518
·97029900	·0·3·0·0·0·0·0·0 ↑	·3015102
·96932871	·0·3·1·0·0·0·0·0 ↑	·3115152
·96835939	·0·3·2·0·0·0·0·0 ↑	·3215202
·96739104	·0·3·3·0·0·0·0·0 ↑	·3315252
·96642362	·0·3·4·0·0·0·0·0 ↑	·3415302
·96545720	·0·3·5·0·0·0·0·0 ↑	·3515352
·96449174	·0·3·6·0·0·0·0·0 ↑	·3615402
·96352725	·0·3·7·0·0·0·0·0 ↑	·3715452
·96256372	·0·3·8·0·0·0·0·0 ↑	·3815502
·96160116	·0·3·9·0·0·0·0·0 ↑	·3915552
·96059601	·0·4·0·0·0·0·0·0 ↑	·4020136
·95963542	·0·4·1·0·0·0·0·0 ↑	·4120186
·95867579	·0·4·2·0·0·0·0·0 ↑	·4220236
·95771710	·0·4·3·0·0·0·0·0 ↑	·4320286
·95675939	·0·4·4·0·0·0·0·0 ↑	·4420336
·95580263	·0·4·5·0·0·0·0·0 ↑	·4520386
·95484683	·0·4·6·0·0·0·0·0 ↑	·4620436
·95389198	·0·4·7·0·0·0·0·0 ↑	·4720486
·95293809	·0·4·8·0·0·0·0·0 ↑	·4820536
·95198515	·0·4·9·0·0·0·0·0 ↑	·4920586
·95099005	·0·5·0·0·0·0·0·0 ↑	·5025170

2 = 69314718   10 = 230258509
4 = 138629436   100 = 460517018
8 = 207944154   1000 = 690775527

N. Nos.	D. Nos.	D. Logs.
·95003906	'0'5'1'0'0'0'0'0 ↑	'5125220
·94908903	'0'5'2'0'0'0'0'0 ↑	'5225270
·94813995	'0'5'3'0'0'0'0'0 ↑	'5325320
·94719182	'0'5'4'0'0'0'0'0 ↑	'5425370
·94624463	'0'5'5'0'0'0'0'0 ↑	'5525420
·94529839	'0'5'6'0'0'0'0'0 ↑	'5625470
·94435310	'0'5'7'0,0'0'0'0 ↑	'5725520
·94340875	'0'5'8'0'0'0'0'0 ↑	'5825570
·94246535	'0'5'9'0'0'0'0'0 ↑	'5925620
·94148015	'0'6'0'0'0'0'0'0 ↑	'6030204
·94053867	'0'6'1'0'0'0'0'0 ↑	'6130254
·93959814	'0'6'2'0'0'0'0'0 ↑	'6230304
·93865855	'0'6'3'0'0'0'0'0 ↑	'6330354
·93771990	'0'6'4'0'0'0'0'0 ↑	'6430404
·93678219	'0'6'5'0'0'0'0'0 ↑	'6530454
·93584541	'0'6'6'0'0'0'0'0 ↑	'6630504
·93490957	'0'6'7'0'0'0'0'0 ↑	'6730554
·93397467	'0'6'8'0'0'0'0'0 ↑	'6830604
·93304070	'0'6'9'0'0'0'0'0 ↑	'6930654
·93206535	'0'7'0'0'0'0'0'0 ↑	'7035238
·93113329	'0'7'1'0'0'0'0'0 ↑	'7135288
·93020216	'0'7'2'0'0'0'0'0 ↑	'7235338
·92927196	'0'7'3'0'0'0'0'0 ↑	'7335388
·92834269	'0'7'4'0'0'0'0'0 ↑	'7435438
·92741435	'0'7'5'0'0'0'0'0 ↑	'7535488
·92648694	'0'7'6'0'0'0'0'0 ↑	'7635538
·92556046	'0'7'7'0'0'0'0'0 ↑	'7735588
·92463490	'0'7'8'0'0'0'0'0 ↑	'7835638
·92371027	'0'7'9'0'0'0'0'0 ↑	'7935688
·92274470	'0'8'0'0'0'0'0'0 ↑	'8040272
·92182196	'0'8'1'0'0'0'0'0 ↑	'8140322
·92090014	'0'8'2'0'0'0'0'0 ↑	'8240372
·91997924	'0'8'3'0'0'0'0'0 ↑	'8340422
·91905927	'0'8'4'0'0'0'0'0 ↑	'8440472
·91814022	'0'8'5'0'0'0'0'0 ↑	'8540522
·91722208	'0'8'6'0'0'0'0'0 ↑	'8640572
·91630486	'0'8'7'0'0'0'0'0 ↑	'8740622
·91538856	'0'8'8'0'0'0'0'0 ↑	'8840672
·91447318	'0'8'9'0'0'0'0'0 ↑	'8940722
·91351725	'0'9'0'0'0'0'0'0 ↑	'9045306
·91260374	'0'9'1'0'0'0'0'0 ↑	'9145356
·91169114	'0'9'2'0'0'0'0'0 ↑	'9245406
·91077945	'0'9'3'0'0'0'0'0 ↑	'9345456
·90986868	'0'9'4'0'0'0'0'0 ↑	'9445506
·90895882	'0'9'5'0'0'0'0'0 ↑	'9545556
·90804987	'0'9'6'0'0'0'0'0 ↑	'9645606
·90714183	'0'9'7'0'0'0'0'0 ↑	'9745656
·90623469	'0'9'8'0'0'0'0'0 ↑	'9845706
·90532846	'0'9'9'0'0'0'0'0 ↑	'9945756
·90000000	'1'0'0'0'0'0'0'0 ↑	'10536052

2= 69314718　　10=230258509
4=138629436　　100=460517018
8=207944154　　1000=690775527

N. Nos.	D. Nos.	D. Logs.
·89910000	'1'0'1'0'0'0'0'0 ↑	'10636102
·89820090	'1'0'2'0'0'0'0'0 ↑	'10736152
·89730270	'1'0'3'0'0'0'0'0 ↑	'10836202
·89640540	'1'0'4'0'0'0'0'0 ↑	'10936252
·89550900	'1'0'5'0'0'0'0'0 ↑	'11036302
·89461350	'1'0'6'0'0'0'0'0 ↑	'11136352
·89371889	'1'0'7'0'0'0'0'0 ↑	'11236402
·89282518	'1'0'8'0'0'0'0'0 ↑	'11336452
·89193235	'1'0'9'0'0'0'0'0 ↑	'11436502
·89100000	'1'1'0'0'0'0'0'0 ↑	'11541086
·89010900	'1'1'1'0'0'0'0'0 ↑	'11641136
·88921890	'1'1'2'0'0'0'0'0 ↑	'11741186
·88832969	'1'1'3'0'0'0'0'0 ↑	'11841236
·88744137	'1'1'4'0'0'0'0'0 ↑	'11941286
·88655393	'1'1'5'0'0'0'0'0 ↑	'12041336
·88566738	'1'1'6'0'0'0'0'0 ↑	'12141386
·88478172	'1'1'7'0'0'0'0'0 ↑	'12241436
·88389694	'1'1'8'0'0'0'0'0 ↑	'12341486
·88301305	'1'1'9'0'0'0'0'0 ↑	'12441536
·88209000	'1'2'0'0'0'0'0'0 ↑	'12546120
·88120791	'1'2'1'0'0'0'0'0 ↑	'12646170
·88032671	'1'2'2'0'0'0'0'0 ↑	'12746220
·87944639	'1'2'3'0'0'0'0'0 ↑	'12846270
·87856695	'1'2'4'0'0'0'0'0 ↑	'12946320
·87768836	'1'2'5'0'0'0'0'0 ↑	'13046370
·87681068	'1'2'6'0'0'0'0'0 ↑	'13146420
·87593386	'1'2'7'0'0'0'0'0 ↑	'13246470
·87505793	'1'2'8'0'0'0'0'0 ↑	'13346520
·87418287	'1'2'9'0'0'0'0'0 ↑	'13446570
·87326910	'1'3'0'0'0'0'0'0 ↑	'13551154
·87239584	'1'3'1'0'0'0'0'0 ↑	'13651204
·87152345	'1'3'2'0'0'0'0'0 ↑	'13751254
·87065193	'1'3'3'0'0'0'0'0 ↑	'13851304
·86978128	'1'3'4'0'0'0'0'0 ↑	'13951354
·86891150	'1'3'5'0'0'0'0'0 ↑	'14051404
·86804259	'1'3'6'0'0'0'0'0 ↑	'14151454
·86717455	'1'3'7'0'0'0'0'0 ↑	'14251504
·86630738	'1'3'8'0'0'0'0'0 ↑	'14351554
·86544108	'1'3'9'0'0'0'0'0 ↑	'14451604
·86453641	'1'4'0'0'0'0'0'0 ↑	'14556188
·86367188	'1'4'1'0'0'0'0'0 ↑	'14656238
·86280821	'1'4'2'0'0'0'0'0 ↑	'14756288
·86194541	'1'4'3'0'0'0'0'0 ↑	'14856338
·86108347	'1'4'4'0'0'0'0'0 ↑	'14956388
·86022239	'1'4'5'0'0'0'0'0 ↑	'15056438
·85936217	'1'4'6'0'0'0'0'0 ↑	'15156488
·85850281	'1'4'7'0'0'0'0'0 ↑	'15256538
·85764431	'1'4'8'0'0'0'0'0 ↑	'15356588
·85678667	'1'4'9'0'0'0'0'0 ↑	'15456638
·85589105	'1'5'0'0'0'0'0'0 ↑	'15561222

2 = 69314718 10 = 230258509
4 = 138629436 100 = 460517018
8 = 207944154 1000 = 690775527

N. Nos.	D. Nos.	D. Logs.
·85503516	1'5'1'0'0'0'0'0 ↑	'15661272
·85418013	1'5'2'0'0'0'0'0 ↑	'15761322
·85332595	1'5'3'0'0'0'0'0 ↑	'15861372
·85247262	1'5'4'0'0'0'0'0 ↑	'15961422
·85162015	1'5'5'0'0'0'0'0 ↑	'16061472
·85076853	1'5'6'0'0'0'0'0 ↑	'16161522
·84991777	1'5'7'0'0'0'0'0 ↑	'16261572
·84906786	1'5'8'0'0'0'0'0 ↑	'16361622
·84821880	1'5'9'0'0'0'0'0 ↑	'16461672
·84733214	'1'6'0'0'0'0'0'0 ↑	'16566256
·84646481	'1'6'1'0'0'0'0'0 ↑	'16666306
·84563832	'1'6'2'0'0'0'0'0 ↑	'16766356
·84479269	'1'6'3'0'0'0'0'0 ↑	'16866406
·84394789	'1'6'4'0'0'0'0'0 ↑	'16966456
·84310394	1'6'5'0'0'0'0'0 ↑	'17066506
·84226084	'1'6'6'0'0'0'0'0 ↑	'17166556
·84141858	1'6'7'0'0'0'0'0 ↑	'17266606
·84057716	1'6'8'0'0'0'0'0 ↑	'17366656
·83973658	1'6'9'0'0'0'0'0 ↑	'17466706
·83885882	'1'7'0'0'0'0'0'0 ↑	'17571290
·83801997	'1'7'1'0'0'0'0'0 ↑	'17671340
·83718196	'1'7'2'0'0'0'0'0 ↑	'17771390
·83634478	'1'7'3'0'0'0'0'0 ↑	'17871440
·83550844	'1'7'4'0'0'0'0'0 ↑	'17971490
·83467294	'1'7'5'0'0'0'0'0 ↑	'18071540
·83383827	'1'7'6'0'0'0'0'0 ↑	'18171590
·83300444	'1'7'7'0'0'0'0'0 ↑	'18271640
·83217144	'1'7'8'0'0'0'0'0 ↑	'18371690
·83133927	'1'7'9'0'0'0'0'0 ↑	'18471740
·83047022	'1'8'0'0'0'0'0'0 ↑	'18576324
·82963975	'1'8'1'0'0'0'0'0 ↑	'18676374
·82881012	'1'8'2'0'0'0'0'0 ↑	'18776424
·82798131	'1'8'3'0'0'0'0'0 ↑	'18876474
·82715333	'1'8'4'0'0'0'0'0 ↑	'18976524
·82632618	'1'8'5'0'0'0'0'0 ↑	'19076574
·82549986	' 8'6'0'0'0'0'0 ↑	'19176624
·82467435	'1'8'7'0'0'0'0'0 ↑	'19276674
·82384968	'1'8'8'0'0'0'0'0 ↑	'19376724
·82302583	'1'8'9'0'0'0'0'0 ↑	'19476774
·82216552	'1'9'0'0'0'0'0'0 ↑	'19581358
·82134336	'1'9'1'0'0'0'0'0 ↑	'19681408
·82052202	'1'9'2'0'0'0'0'0 ↑	'19781458
·81970150	'1'9'3'0'0'0'0'0 ↑	'19881508
·81888180	'1'9'4'0'0'0'0'0 ↑	'19981558
·81806292	'1'9'5'0'0'0'0'0 ↑	'20081608
·81724486	'1'9'6'0'0'0'0'0 ↑	'20181658
·81642762	'1'9'7'0'0'0'0'0 ↑	'20281708
·81561120	'1'9'8'0'0'0'0'0 ↑	'20381758
·81479559	'1'9'9'0'0'0'0'0 ↑	'20481808
·81000000	'2'0'0'0'0'0'0'0 ↑	'21072103

2= 69314718 10=230258509
4=138629436 100=460517018
8=207944154 1000=690775527

N. Nos.	D. Nos.	D. Logs.
·80919000	'2·0·1·0·0·0·0·0 ↑	'21172153
·80838081	'2·0·2·0·0·0·0·0 ↑	'21272203
·80757243	'2·0·3·0·0·0·0·0 ↑	'21372253
·80676486	'2·0·4·0·0·0·0·0 ↑	'21472303
·80595810	'2·0·5·0·0·0·0·0 ↑	'21572353
·80515215	'2·0·6·0·0·0·0·0 ↑	'21672403
·80434698	'2·0·7·0·0·0·0·0 ↑	'21772453
·80354264	'2·0·8·0·0·0·0·0 ↑	'21872503
·80273909	'2·0·9·0·0·0·0·0 ↑	'21972553
·80190000	'2·1·0·0·0·0·0·0 ↑	'22077137
·80109810	'2·1·1·0·0·0·0·0 ↑	'22177187
·80029701	'2·1·2·0·0·0·0·0 ↑	'22277237
·79949671	'2·1·3·0·0·0·0·0 ↑	'22377287
·79869722	'2·1·4·0·0·0·0·0 ↑	'22477337
·79789853	'2·1·5·0·0·0·0·0 ↑	'22577387
·79710064	'2·1·6·0·0·0·0·0 ↑	'22677437
·79630354	'2·1·7·0·0·0·0·0 ↑	'22777487
·79550724	'2·1·8·0·0·0·0·0 ↑	'22877537
·79471174	'2·1·9·0·0·0·0·0 ↑	'22977587
·79388100	'2·2·0·0·0·0·0·0 ↑	'23082171
·79308712	'2·2·1·0·0·0·0·0 ↑	'23182221
·79229404	'2·2·2·0·0·0·0·0 ↑	'23282271
·79150174	'2·2·3·0·0·0·0·0 ↑	'23382321
·79071024	'2·2·4·0·0·0·0·0 ↑	'23482371
·78991953	'2·2·5·0·0·0·0·0 ↑	'23582421
·78912961	'2·2·6·0·0·0·0·0 ↑	'23682471
·78834048	'2·2·7·0·0·0·0·0 ↑	'23782521
·78755214	'2·2·8·0·0·0·0·0 ↑	'23882571
·78676458	'2·2·9·0·0·0·0·0 ↑	'23982621
·78594219	'2·3·0·0·0·0·0·0 ↑	'24087205
·78515625	'2·3·1·0·0·0·0·0 ↑	'24187255
·78437110	'2·3·2·0·0·0·0·0 ↑	'24287305
·78358673	'2·3·3·0·0·0·0·0 ↑	'24387355
·78280315	'2·3·4·0·0·0·0·0 ↑	'24487405
·78202035	'2·3·5·0·0·0·0·0 ↑	'24587455
·78123833	'2·3·6·0·0·0·0·0 ↑	'24687505
·78045710	'2·3·7·0·0·0·0·0 ↑	'24787555
·77967665	'2·3·8·0·0·0·0·0 ↑	'24887605
·77889698	'2·3·9·0·0·0·0·0 ↑	'24987655
·77808277	'2·4·0·0·0·0·0·0 ↑	'25092239
·77730469	'2·4·1·0·0·0·0·0 ↑	'25192289
·77652739	'2·4·2·0·0·0·0·0 ↑	'25292339
·77575087	'2·4·3·0·0·0·0·0 ↑	'25392389
·77497512	'2·4·4·0·0·0·0·0 ↑	'25492439
·77420015	'2·4·5·0·0·0·0·0 ↑	'25592489
·77342595	'2·4·6·0·0·0·0·0 ↑	'25692539
·77265253	'2·4·7·0·0·0·0·0 ↑	'25792589
·77187988	'2·4·8·0·0·0·0·0 ↑	'25892639
·77110801	'2·4·9·0·0·0·0·0 ↑	'25992689
·77030194	'2·5·0·0·0·0·0·0 ↑	'26097273

2 = 69314718	10 = 230258509
4 = 138629436	100 = 460517018
8 = 207944154	1000 = 690775527

N. Nos.	D. Nos.	D. Logs.
·76953164	'2'5'1'0'0'0'0'0 ↑	'26197323
·76876211	'2'5'2'0'0'0'0'0 ↑	'26297373
·76799335	'2'5'3'0'0'0'0'0 ↑	'26397423
·76722536	'2'5'4'0'0'0'0'0 ↑	'26497473
·76645814	'2'5'5'0'0'0'0'0 ↑	'26597523
·76569169	'2'5'6'0'0'0'0'0 ↑	'26697573
·76492600	'2'5'7'0,0'0'0'0 ↑	'26797623
·76416108	'2'5'8'0'0'0'0'0 ↑	'26897673
·76349692	'2'5'9'0'0'0'0'0 ↑	'26997723
·76259892	'2'6'0'0'0'0'0'0 ↑	'27102307
·76183633	'2'6'1'0'0'0'0'0 ↑	'27202357
·76107449	'2'6'2'0'0'0'0'0 ↑	'27302407
·76031341	'2'6'3'0'0'0'0'0 ↑	'27402457
·75955310	'2'6'4'0'0'0'0'0 ↑	'27502507
·75879355	'2'6'5'0'0'0'0'0 ↑	'27602557
·75803457	'2'6'6'0'0'0'0'0 ↑	'27702607
·75727672	'2'6'7'0'0'0'0'0 ↑	'27802657
·75651944	'2'6'8'0'0'0'0'0 ↑	'27902707
·75576292	'2'6'9'0'0'0'0'0 ↑	'28002757
·75497293	'2'7'0'0'0'0'0'0 ↑	'28107341
·75421796	'2'7'1'0'0'0'0'0 ↑	'28207391
·75346374	'2'7'2'0'0'0'0'0 ↑	'28307441
·75271028	'2'7'3'0'0'0'0'0 ↑	'28407491
·75195757	'2'7'4'0'0'0'0'0 ↑	'28507541
·75120561	'2'7'5'0'0'0'0'0 ↑	'28607591
·75045441	'2'7'6'0'0'0'0'0 ↑	'28707641
·74970396	'2'7'7'0'0'0'0'0 ↑	'28807691
·74895426	'2'7'8'0'0'0'0'0 ↑	'28907741
·74820531	'2'7'9'0'0'0'0'0 ↑	'29007791
·74742320	'2'8'0'0'0'0'0'0 ↑	'29112375
·74667578	'2'8'1'0'0'0'0'0 ↑	'29212425
·74592911	'2'8'2'0'0'0'0'0 ↑	'29312475
·74518318	'2'8'3'0'0'0'0'0 ↑	'29412525
·74443800	'2'8'4'0'0'0'0'0 ↑	'29512575
·74369356	'2'8'5'0'0'0'0'0 ↑	'29612625
·74294987	'2'8'6'0'0'0'0'0 ↑	'29712675
·74220692	'2'8'7'0'0'0'0'0 ↑	'29812725
·74146471	'2'8'8'0'0'0'0'0 ↑	'29912775
·74072325	'2'8'9'0'0'0'0'0 ↑	'30012825
·73994897	'2'9'0'0'0'0'0'0 ↑	'30117409
·73920903	'2'9'1'0'0'0'0'0 ↑	'30217459
·73846983	'2'9'2'0'0'0'0'0 ↑	'30317509
·73773137	'2'9'3'0'0'0'0'0 ↑	'30417559
·73699364	'2'9'4'0'0'0'0'0 ↑	'30517609
·73625665	'2'9'5'0'0'0'0'0 ↑	'30617659
·73552040	'2'9'6'0'0'0'0'0 ↑	'30717709
·73478488	'2'9'7'0'0'0'0'0 ↑	'30817759
·73405010	'2'9'8'0'0'0'0'0 ↑	'30917809
·73331605	'2'9'9'0'0'0'0'0 ↑	'31017859
·72900000	'3'0'0'0'0'0'0'0 ↑	'31608155

2 = 69314718   10 = 230258509
4 = 138629436   100 = 460517018
8 = 207944154   1000 = 690775527

N. Nos.	D. Nos.	D. Logs.
·72827100	'3'0'1'0'0'0'0'0 ↑	'31708205
·72754273	'3'0'2'0'0'0'0'0 ↑	'31808255
·72681519	'3'0'3'0'0'0'0'0 ↑	'31908305
·72608837	'3'0'4'0'0'0'0'0 ↑	'32008355
·72536228	'3'0'5'0'0'0'0'0 ↑	'32108405
·72463692	'3'0'6'0'0'0'0'0 ↑	'32208455
·72391228	'3'0'7'0'0'0'0'0 ↑	'32308505
·72318837	'3'0'8'0'0'0'0'0 ↑	'32408555
·72246518	'3'0'9'0'0'0'0'0 ↑	'32508605
·72171000	'3'1'0'0'0'0'0'0 ↑	'32613189
·72098829	'3'1'1'0'0'0'0'0 ↑	'32713239
·72026731	'3'1'2'0'0'0'0'0 ↑	'32813289
·71954705	'3'1'3'0'0'0'0'0 ↑	'32913339
·71882751	'3'1'4'0'0'0'0'0 ↑	'33013389
·71810869	'3'1'5'0'0'0'0'0 ↑	'33113439
·71739059	'3'1'6'0'0'0'0'0 ↑	'33213489
·71667320	'3'1'7'0'0'0'0'0 ↑	'33313539
·71595653	'3'1'8'0'0'0'0'0 ↑	'33413589
·71524057	'3'1'9'0'0'0'0'0 ↑	'33513639
·71449290	'3'2'0'0'0'0'0'0 ↑	'33618223
·71377841	'3'2'1'0'0'0'0'0 ↑	'33718273
·71306464	'3'2'2'0'0'0'0'0 ↑	'33818323
·71235158	'3'2'3'0'0'0'0'0 ↑	'33918373
·71163923	'3'2'4'0'0'0'0'0 ↑	'34018423
·71092760	'3'2'5'0'0'0'0'0 ↑	'34118473
·71021668	'3'2'6'0'0'0'0'0 ↑	'34218523
·70950647	'3'2'7'0'0'0'0'0 ↑	'34318573
·70879697	'3'2'8'0'0'0'0'0 ↑	'34418623
·70808818	'3'2'9'0'0'0'0'0 ↑	'34518673
·70734797	'3'3'0'0'0'0'0'0 ↑	'34623257
·70664063	'3'3'1'0'0'0'0'0 ↑	'34723307
·70593399	'3'3'2'0'0'0'0'0 ↑	'34823357
·70522806	'3'3'3'0'0'0'0'0 ↑	'34923407
·70452284	'3'3'4'0'0'0'0'0 ↑	'35023457
·70381832	'3'3'5'0'0'0'0'0 ↑	'35123507
·70311451	'3'3'6'0'0'0'0'0 ↑	'35223557
·70241140	'3'3'7'0'0'0'0'0 ↑	'35323607
·70170899	'3'3'8'0'0'0'0'0 ↑	'35423657
·70100729	'3'3'9'0'0'0'0'0 ↑	'35523707
·70027449	'3'4'0'0'0'0'0'0 ↑	'35628291
·69957422	'3'4'1'0'0'0'0'0 ↑	'35728341
·69887465	'3'4'2'0'0'0'0'0 ↑	'35828391
·69817578	'3'4'3'0'0'0'0'0 ↑	'35928441
·69747761	'3'4'4'0'0'0'0'0 ↑	'36028491
·69678014	'3'4'5'0'0'0'0'0 ↑	'36128541
·69608336	'3'4'6'0'0'0'0'0 ↑	'36228591
·69538728	'3'4'7'0'0'0'0'0 ↑	'36328641
·69469190	'3'4'8'0'0'0'0'0 ↑	'36428691
·69399721	'3'4'9'0'0'0'0'0 ↑	'36528741
·69327175	'3'5'0'0'0'0'0'0 ↑	'36633325

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N. Nos.	D. Nos.	D. Logs.
·69257848	·35100000 ↑	·36733375
·69188591	·35200000 ↑	·36833425
·69119403	·35300000 ↑	·36933475
·69050284	·35400000 ↑	·37033525
·68981234	·35500000 ↑	·37133575
·68912253	·35600000 ↑	·37233625
·68843339	·35700000 ↑	·37333675
·68774496	·35800000 ↑	·37433725
·68705722	·35900000 ↑	·37533775
·68633902	·360000000 ↑	·37638359
·68565269	·36100000 ↑	·37738409
·68496704	·36200000 ↑	·37838459
·68428208	·36300000 ↑	·37938509
·68359780	·36400000 ↑	·38038559
·68291421	·36500000 ↑	·38138609
·68223130	·36600000 ↑	·38238659
·68154907	·36700000 ↑	·38338709
·68086753	·36800000 ↑	·38438759
·68018667	·36900000 ↑	·38538809
·67947563	·370000000 ↑	·38643393
·67879616	·37100000 ↑	·38743443
·67811737	·37200000 ↑	·38843493
·67743926	·37300000 ↑	·38943543
·67676183	·37400000 ↑	·39043593
·67608507	·37500000 ↑	·39143643
·67540899	·37600000 ↑	·39243693
·67473359	·37700000 ↑	·39343743
·67405881	·37800000 ↑	·39443793
·67338475	·37900000 ↑	·39543843
·67268087	·380000000 ↑	·39648427
·67200819	·38100000 ↑	·39748477
·67133619	·38200000 ↑	·39848527
·67066486	·38300000 ↑	·39948577
·66999420	·38400000 ↑	·40048627
·66932421	·38500000 ↑	·40148677
·66865489	·38600000 ↑	·40248727
·66798624	·38700000 ↑	·40348777
·66731826	·38800000 ↑	·40448827
·66665095	·38900000 ↑	·40548877
·66595406	·390000000 ↑	·40653461
·66528811	·39100000 ↑	·40753511
·66462283	·39200000 ↑	·40853561
·66395821	·39300000 ↑	·40953611
·66329426	·39400000 ↑	·41053661
·66263097	·39500000 ↑	·41153711
·66196834	·39600000 ↑	·41253761
·66130638	·39700000 ↑	·41353811
·66064508	·39800000 ↑	·41453861
·65998444	·39900000 ↑	·41553911
·65610000	·400000000 ↑	·42144206

2 = 69314718   10 = 230258509
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N. Nos.	D. Nos.	D. Logs.
·65544330	'4·0100000 ↑	'42244256
·65478846	'4·0200000 ↑	'42344306
·65413368	'4·0300000 ↑	'42444356
·65347955	'4·0400000 ↑	'42544406
·65282607	'4·0500000 ↑	'42644456
·65217325	'4·0600000 ↑	'42744506
·65152005	'4·0700000 ↑	'42844556
·65086853	'4·0800000 ↑	'42944606
·65021869	'4·0900000 ↑	'43044656
·64953900	'4·1000000 ↑	'43149240
·64888947	'4·1100000 ↑	'43249290
·64824059	'4·1200000 ↑	'43349340
·64759235	'4·1300000 ↑	'43449390
·64694476	'4·1400000 ↑	'43549440
·64629780	'4·1500000 ↑	'43649490
·64565149	'4·1600000 ↑	'43749540
·64500585	'4·1700000 ↑	'43849590
·64436084	'4·1800000 ↑	'43949640
·64371648	'4·1900000 ↑	'44049690
·64304361	'4·2000000 ↑	'44154274
·64240057	'4·2100000 ↑	'44254324
·64175817	'4·2200000 ↑	'44354374
·64111642	'4·2300000 ↑	'44454424
·64047531	'4·2400000 ↑	'44554474
·63983484	'4·2500000 ↑	'44654524
·63919501	'4·2600000 ↑	'44754574
·63855578	'4·2700000 ↑	'44854624
·63791723	'4·2800000 ↑	'44954674
·63727931	'4·2900000 ↑	'45054724
·63661317	'4·3000000 ↑	'45159308
·63597656	'4·3100000 ↑	'45259358
·63534059	'4·3200000 ↑	'45359408
·63470525	'4·3300000 ↑	'45459458
·63407055	'4·3400000 ↑	'45559508
·63343648	'4·3500000 ↑	'45659558
·63280305	'4·3600000 ↑	'45759608
·63217025	'4·3700000 ↑	'45859658
·63153808	'4·3800000 ↑	'45959708
·63090655	'4·3900000 ↑	'46059758
·63024704	'4·4000000 ↑	'46164342
·62961680	'4·4100000 ↑	'46264392
·62898719	'4·4200000 ↑	'46364442
·62835821	'4·4300000 ↑	'46464492
·62762986	'4·4400000 ↑	'46564542
·62710210	'4·4500000 ↑	'46664592
·62647500	'4·4600000 ↑	'46764642
·62584853	'4·4700000 ↑	'46864692
·62522268	'4·4800000 ↑	'46964742
·62459745	'4·4900000 ↑	'47064792
·62394457	'4·5000000 ↑	'47169376

2 = 69314718   10 = 230258509
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8 = 207944154   1000 = 690775527

N. Nos.	D. Nos.	D. Logs.
·62332063	'4'5'1'0'0'0'0'0 ↑	'47269426
·62269731	'4'5'2'0'0'0'0'0 ↑	'47369476
·62207461	'4'5'3'0'0'0'0'0 ↑	'47469526
·62145254	'4'5'4'0'0'0'0'0 ↑	'47569576
·62083109	'4'5'5'0'0'0'0'0 ↑	'47669626
·62021026	'4'5'6'0'0'0'0'0 ↑	'47769676
·61959005	'4'5'7'0'0'0'0'0 ↑	'47869726
·61897046	'4'5'8'0'0'0'0'0 ↑	'47969776
·61835149	'4'5'9'0'0'0'0'0 ↑	'48069826
·61770512	'4'6'0'0'0'0'0'0 ↑	'48174410
·61708742	'4'6'1'0'0'0'0'0 ↑	'48274460
·61647034	'4'6'2'0'0'0'0'0 ↑	'48374510
·61585386	'4'6'3'0'0'0'0'0 ↑	'48474560
·61523801	'4'6'4'0'0'0'0'0 ↑	'48574610
·61462277	'4'6'5'0'0'0'0'0 ↑	'48674660
·61400815	'4'6'6'0'0'0'0'0 ↑	'48774710
·61339414	'4'6'7'0'0'0'0'0 ↑	'48874769
·61278075	'4'6'8'0'0'0'0'0 ↑	'48974810
·61216797	'4'6'9'0'0'0'0'0 ↑	'49074860
·61152807	'4'7'0'0'0'0'0'0 ↑	'49179444
·61091655	'4'7'1'0'0'0'0'0 ↑	'49279494
·61030564	'4'7'2'0'0'0'0'0 ↑	'49379544
·60969534	'4'7'3'0'0'0'0'0 ↑	'49479594
·60908565	'4'7'4'0'0'0'0'0 ↑	'49579644
·60847657	'4'7'5'0'0'0'0'0 ↑	'49679694
·60786807	'4'7'6'0'0'0'0'0 ↑	'49779744
·60726020	'4'7'7'0'0'0'0'0 ↑	'49879794
·60665294	'4'7'8'0'0'0'0'0 ↑	'49979844
·60604629	'4'7'9'0'0'0'0'0 ↑	'50079894
·60541279	'4'8'0'0'0'0'0'0 ↑	'50184478
·60480738	'4'8'1'0'0'0'0'0 ↑	'50284528
·60420258	'4'8'2'0'0'0'0'0 ↑	'50384578
·60359838	'4'8'3'0'0'0'0'0 ↑	'50484628
·60299479	'4'8'4'0'0'0'0'0 ↑	'50584678
·60239180	'4'8'5'0'0'0'0'0 ↑	'50684728
·60178941	'4'8'6'0'0'0'0'0 ↑	'50784778
·60118763	'4'8'7'0'0'0'0'0 ↑	'50884828
·60058641	'4'8'8'0'0'0'0'0 ↑	'50984878
·59998583	'4'8'9'0'0'0'0'0 ↑	'51084928
·59935866	'4'9'0'0'0'0'0'0 ↑	'51189512
·59875931	'4'9'1'0'0'0'0'0 ↑	'51289562
·59816056	'4'9'2'0'0'0'0'0 ↑	'51389612
·59756240	'4'9'3'0'0'0'0'0 ↑	'51489662
·59696484	'4'9'4'0'0'0'0'0 ↑	'51589712
·59636788	'4'9'5'0'0'0'0'0 ↑	'51689762
·59577152	'4'9'6'0'0'0'0'0 ↑	'51789812
·59517572	'4'9'7'0'0'0'0'0 ↑	'51889862
·59458055	'4'9'8'0'0'0'0'0 ↑	'51989912
·59398597	'4'9'9'0'0'0'0'0 ↑	'52089962
·59049000	'5'0'0'0'0'0'0'0 ↑	'52680258

2 = 69314718	10 = 230258509
4 = 138629436	100 = 460517018
8 = 207944154	1000 = 690775527

N. Nos.	D. Nos.	D. Logs.
·58989951	'5'0'1'0'0'0'0'0 ↑	'52780308
·58930962	'5'0'2'0'0'0'0'0 ↑	'52880358
·58872032	'5'0'3'0'0'0'0'0 ↑	'52980408
·58813160	'5'0'4'0'0'0'0'0 ↑	'53080458
·58754347	'5'0'5'0'0'0'0'0 ↑	'531805 8
·58695593	'5'0'6'0'0'0'0'0 ↑	'53280558
·58636898	'5'0'7'0'0'0'0'0 ↑	'53380608
·58578258	'5'0'8'0'0'0'0'0 ↑	'53480658
·58519680	'5'0'9'0'0'0'0'0 ↑	'53580708
·58458510	'5'1'0'0'0'0'0'0 ↑	'53685292
·58400052	'5'1'1'0'0'0'0'0 ↑	'53785342
·58341652	'5'1'2'0'0'0'0'0 ↑	'53885392
·58283311	'5'1'3'0'0'0'0'0 ↑	'53985442
·58225026	'5'1'4'0'0'0'0'0 ↑	'54085492
·58166801	'5'1'5'0'0'0'0'0 ↑	'54185542
·58108635	'5'1'6'0'0'0'0'0 ↑	'54285592
·58050527	'5'1'7'0'0'0'0'0 ↑	'54385642
·57992476	'5'1'8'0'0'0'0'0 ↑	'54485692
·57934483	'5'1'9'0'0'0'0'0 ↑	'54585742
·57873925	'5'2'0'0'0'0'0'0 ↑	'54690326
·57816052	'5'2'1'0'0'0'0'0 ↑	'54790376
·57758236	'5'2'2'0'0'0'0'0 ↑	'54890426
·57700478	'5'2'3'0'0'0'0'0 ↑	'54990476
·57642778	'5'2'4'0'0'0'0'0 ↑	'55090526
·57585136	'5'2'5'0'0'0'0'0 ↑	'55190576
·57527551	'5'2'6'0'0'0'0'0 ↑	'55290626
·57470021	'5'2'7'0'0'0'0'0 ↑	'55390676
·57412551	'5'2'8'0'0'0'0'0 ↑	'55490726
·57355138	'5'2'9'0'0'0'0'0 ↑	'55590776
·57295186	'5'3'0'0'0'0'0'0 ↑	'55695360
·57237891	'5'3'1'0'0'0'0'0 ↑	'55795410
·57180653	'5'3'2'0'0'0'0'0 ↑	'55895460
·57123472	'5'3'3'0'0'0'0'0 ↑	'55995510
·57066349	'5'3'4'0'0'0'0'0 ↑	'56095560
·57009283	'5'3'5'0'0'0'0'0 ↑	'56195610
·56952274	'5'3'6'0'0'0'0'0 ↑	'56295660
·56895322	'5'3'7'0'0'0'0'0 ↑	'56395710
·56838427	'5'3'8'0'0'0'0'0 ↑	'56495760
·56781589	'5'3'9'0'0'0'0'0 ↑	'56595810
·56722234	'5'4'0'0'0'0'0'0 ↑	'56700394
·56665512	'5'4'1'0'0'0'0'0 ↑	'56800444
·56608847	'5'4'2'0'0'0'0'0 ↑	'56900494
·56552239	'5'4'3'0'0'0'0'0 ↑	'57000544
·56495687	'5'4'4'0'0'0'0'0 ↑	'57100594
·56439189	'5'4'5'0'0'0'0'0 ↑	'57200644
·56382752	'5'4'6'0'0'0'0'0 ↑	'57300694
·56326367	'5'4'7'0'0'0'0'0 ↑	'57400744
·56270041	'5'4'8'0'0'0'0'0 ↑	'57500794
·56213771	'5'4'9'0'0'0'0'0 ↑	'57600844
·56155012	'5'5'0'0'0'0 0 ↑	'57705428

2 = 69314718   10 = 230258509
4 = 138629436   100 = 460517018
8 = 207944154   1000 = 690775527

N. Nos.	D. Nos.	D. Logs.
'56098857	'5'5'1'0'0'0'0'0 ↑	'57805478
'56042758	'5'5'2'0'0'0'0'0 ↑	'57905528
'55986715	'5'5'3'0'0'0'0'0 ↑	'58005578
'55930728	'5'5'4'0'0'0'0'0 ↑	'58105628
'55874797	'5'5'5'0'0'0'0'0 ↑	'58205678
'55818922	'5'5'6'0'0'0'0'0 ↑	'58305728
'55763103	'5'5'7'0'0'0'0'0 ↑	'58405778
'55707340	'5'5'8'0'0'0'0'0 ↑	'58505828
'55651633	'5'5'9'0'0'0'0'0 ↑	'58605878
'55593462	'5'6'0'0'0'0'0'0 ↑	'58710462
'55537869	'5'6'1'0'0'0'0'0 ↑	'58810512
'55482331	'5'6'2'0'0'0'0'0 ↑	'58910562
'55426849	'5'6'3'0'0'0'0'0 ↑	'59010612
'55371422	'5'6'4'0'0'0'0'0 ↑	'59110662
'55316051	'5'6'5'0'0'0'0'0 ↑	'59210712
'55260735	'5'6'6'0'0'0'0'0 ↑	'59310762
'55205474	'5'6'7'0'0'0'0'0 ↑	'59410812
'55150269	'5'6'8'0'0'0'0'0 ↑	'59510862
'55095119	'5'6'9'0'0'0'0'0 ↑	'59610912
'55037527	'5'7'0'0'0'0'0'0 ↑	'59715496
'54982489	'5'7'1'0'0'0'0'0 ↑	'59815546
'54927507	'5'7'2'0'0'0'0'0 ↑	'59915596
'54872579	'5'7'3'0'0'0'0'0 ↑	'60015646
'54817706	'5'7'4'0'0'0'0'0 ↑	'60115696
'54762888	'5'7'5'0'0'0'0'0 ↑	'60215746
'54708125	'5'7'6'0'0'0'0'0 ↑	'60315796
'54653417	'5'7'7'0'0'0'0'0 ↑	'60415846
'54598764	'5'7'8'0'0'0'0'0 ↑	'60515896
'54544165	'5'7'9'0'0'0'0'0 ↑	'60615946
'54487152	'5'8'0'0'0'0'0'0 ↑	'60720530
'54432665	'5'8'1'0'0'0'0'0 ↑	'60820580
'54378232	'5'8'2'0'0'0'0'0 ↑	'60920630
'54323854	'5'8'3'0'0'0'0'0 ↑	'61020680
'54269530	'5'8'4'0'0'0'0'0 ↑	'61120730
'54215260	'5'8'5'0'0'0'0'0 ↑	'61220780
'54161045	'5'8'6'0'0'0'0'0 ↑	'61320830
'54106884	'5'8'7'0'0'0'0'0 ↑	'61420880
'54052777	'5'8'8'0'0'0'0'0 ↑	'61520930
'53998724	'5'8'9'0'0'0'0'0 ↑	'61620980
'53942280	5'9'0'0'0'0'0'0 ↑	'61725564
'53888338	'5'9'1'0'0'0'0'0 ↑	'61825614
'53834450	'5'9'2'0'0'0'0'0 ↑	'61925664
'53780616	'5'9'3'0'0'0'0'0 ↑	'62025714
'53726835	5'9'4'0'0'0'0'0 ↑	'62125764
'53673108	'5'9'5'0'0'0'0'0 ↑	'62225814
'53619435	'5'9'6'0'0'0'0'0 ↑	'62325864
'53565816	'5'9'7'0'0'0'0'0 ↑	'62425914
'53512250	'5'9'8'0'0'0'0'0 ↑	'62525964
'53458739	'5'9'9'0'0'0'0'0 ↑	'62626014
'53144100	'6'0'0'0'0'0'0'0 ↑	'63216309

2 = 69314718   10 = 230258509
4 = 138629436   100 = 460517018
8 = 207944154   1000 = 690775527

N. Nos.	D. Nos.	D. Logs.
·53090956	·601·00000 ↑	·63316359
·53037865	·602·00000 ↑	·63416409
·52984827	·603·00000 ↑	·63516459
·52931842	·604·00000 ↑	·63616509
·52878910	·605·00000 ↑	·63716559
·52826031	·606·00000 ↑	·63816609
·52773205	·607·00000 ↑	·63916659
·52720432	·608·00000 ↑	·64016709
·52667712	·609·00000 ↑	·64116759
·52612659	·61·000000 ↑	·64221343
·52562046	·611·00000 ↑	·64321393
·52507484	·612·00000 ↑	·64421443
·52454979	·613·00000 ↑	·64521493
·52402524	·614·00000 ↑	·64621543
·52350121	·615·00000 ↑	·64721593
·52297771	·616·00000 ↑	·64821643
·52245475	·617·00000 ↑	·64921693
·52193228	·618·00000 ↑	·65021743
·52141035	·619·00000 ↑	·65121793
·52086532	·62·0000000 ↑	·65226377
·52034445	·621·00000 ↑	·65326427
·51982402	·622·00000 ↑	·65426477
·51930419	·623·00000 ↑	·65526527
·51878489	·624·00000 ↑	·65626577
·51826610	·625·00000 ↑	·65726627
·51774784	·626·00000 ↑	·65826677
·51723009	·627·00000 ↑	·65926727
·51671286	·628·00000 ↑	·66026777
·51619614	·629·00000 ↑	·66126827
·51565667	·63·0000000 ↑	·66231411
·51514101	·631·00000 ↑	·66331461
·51462587	·632·00000 ↑	·66431511
·51411124	·633·00000 ↑	·66531561
·51359713	·634·00000 ↑	·66631611
·51308353	·635·00000 ↑	·66731661
·51257045	·636·00000 ↑	·66831711
·51205788	·637·00000 ↑	·66931761
·51154582	·638·00000 ↑	·67031811
·51103428	·639·00000 ↑	·67131861
·51050010	·64·0000000 ↑	·67236445
·50998960	·641·00000 ↑	·67336495
·50947961	·642·00000 ↑	·67436545
·50897013	·643·00000 ↑	·67536595
·50846116	·644·00000 ↑	·67636645
·50795270	·645·00000 ↑	·67736695
·50744475	·646·00000 ↑	·67836745
·50693731	·647·00000 ↑	·67936795
·50643037	·648·00000 ↑	·68036845
·50592395	·649·00000 ↑	·68136895
·50539510	·65·0000000 ↑	·68241479

2 = 69314718 10 = 230258509
4 = 138629436 100 = 460517018
8 = 207944154 1000 = 690775527

N. Nos.	D. Nos.	D. Logs.
·50488970	·65·1·0·0·0·0·0 ↑	·68341529
·50438481	·65·2·0·0·0·0·0 ↑	·68441579
·50388043	·65·3·0·0·0·0·0 ↑	·68541629
·50337655	·65·4·0·0·0·0·0 ↑	·68641679
·50287317	·65·5·0·0·0·0·0 ↑	·68741729
·50237030	·65·6·0·0·0·0·0 ↑	·68841779
·50186793	·65·7·0·0·0·0·0 ↑	·68941829
·50136606	·65·8·0·0·0·0·0 ↑	·69041879
·50086470	·65·9·0·0·0·0·0 ↑	·69141929
·50034115	·66·0·0·0·0·0·0 ↑	·69246513
·49984081	·66·1·0·0·0·0·0 ↑	·69346563
·49934097	·66·2·0·0·0·0·0 ↑	·69446613
·49884163	·66·3·0·0·0·0·0 ↑	·69546663
·49834279	·66·4·0·0·0·0·0 ↑	·69646713
·49784445	·66·5·0·0·0·0·0 ↑	·69746763
·49734661	·66·6·0·0·0·0·0 ↑	·69846813
·49684926	·66·7·0·0·0·0·0 ↑	·69946863
·49635241	·66·8·0·0·0·0·0 ↑	·70046913
·49585606	·66·9·0·0·0·0·0 ↑	·70146963
·49533774	·67·0·0·0·0·0·0 ↑	·70251547
·49484240	·67·1·0·0·0·0·0 ↑	·70351597
·49434756	·67·2·0·0·0·0·0 ↑	·70451647
·49385321	·67·3·0·0·0·0·0 ↑	·70551697
·49335936	·67·4·0·0·0·0·0 ↑	·70651747
·49286600	·67·5·0·0·0·0·0 ↑	·70751797
·49237313	·67·6·0·0·0·0·0 ↑	·70851847
·49188076	·67·7·0·0·0·0·0 ↑	·70951897
·49138888	·67·8·0·0·0·0·0 ↑	·71051947
·49089749	·67·9·0·0·0·0·0 ↑	·71151997
·49038436	·68·0·0·0·0·0·0 ↑	·71256581
·48989398	·68·1·0·0·0·0·0 ↑	·71356631
·48940409	·68·2·0·0·0·0·0 ↑	·71456681
·48891469	·68·3·0·0·0·0·0 ↑	·71556731
·48842578	·68·4·0·0·0·0·0 ↑	·71656781
·48793735	·68·5·0·0·0·0·0 ↑	·71756831
·48744941	·68·6·0·0·0·0·0 ↑	·71856881
·48696196	·68·7·0·0·0·0·0 ↑	·71956931
·48647500	·68·8·0·0·0·0·0 ↑	·72056981
·48598852	·68·9·0·0·0·0·0 ↑	·72157031
·48548052	·69·0·0·0·0·0·0 ↑	·72261615
·48499504	·69·1·0·0·0·0·0 ↑	·72361665
·48451004	·69·2·0·0·0·0·0 ↑	·72461715
·48402553	·69·3·0·0·0·0·0 ↑	·72561765
·48354150	·69·4·0·0·0·0·0 ↑	·72661815
·48305796	·69·5·0·0·0·0·0 ↑	·72761865
·48257490	·69·6·0·0·0·0·0 ↑	·72861915
·48209233	·69·7·0·0·0·0·0 ↑	·72961965
·48161024	·69·8·0·0·0·0·0 ↑	·73062015
·48112863	·69·9·0·0·0·0·0 ↑	·73162065
·47829690	·7·0·0·0·0·0·0·0 ↑	·73752361

2 = 69314718 10 = 230258509
4 = 138629436 100 = 460517018
8 = 207944154 1000 = 690775527

N. Nos.	D. Nos.	D. Logs.
·47781860	'7'0'1'0'0'0'0'0 ↑	'73852411
·47734078	'7'0'2'0'0'0'0'0 ↑	'73952461
·47686344	'7'0'3'0'0'0'0'0 ↑	'74052511
·47638658	'7'0'4'0'0'0'0'0 ↑	'74152561
·47591019	'7'0'5'0'0'0'0'0 ↑	'74252611
·47543428	'7'0'6'0'0'0'0'0 ↑	'74352661
·47495885	'7'0'7'0'0'0'0'0 ↑	'74452711
·47448389	'7'0'8'0'0'0'0'0 ↑	'74552761
·47400941	'7'0'9'0'0'0'0'0 ↑	'74652811
·47351393	'7'1'0'0'0'0'0'0 ↑	'74757395
·47304042	'7'1'1'0'0'0'0'0 ↑	'74857445
·47256738	'7'1'2'0'0'0'0'0 ↑	'74957495
·47209481	'7'1'3'0'0'0'0'0 ↑	'75057545
·47162272	'7'1'4'0'0'0'0'0 ↑	'75157595
·47115109	'7'1'5'0'0'0'0'0 ↑	'75257645
·47067994	'7'1'6'0'0'0'0'0 ↑	'75357695
·47020926	'7'1'7'0'0'0'0'0 ↑	'75457745
·46973905	'7'1'8'0'0'0'0'0 ↑	'75557795
·46926931	'7'1'9'0'0'0'0'0 ↑	'75657845
·46877879	'7'2'0'0'0'0'0'0 ↑	'75762429
·46831001	'7'2'1'0'0'0'0'0 ↑	'75862479
·46784170	'7'2'2'0'0'0'0'0 ↑	'75962529
·46737386	'7'2'3'0'0'0'0'0 ↑	'76062579
·46690649	'7'2'4'0'0'0'0'0 ↑	'76162629
·46643958	'7'2'5'0'0'0'0'0 ↑	'76262679
·46597314	'7'2'6'0'0'0'0'0 ↑	'76362729
·46550717	'7'2'7'0'0'0'0'0 ↑	'76462779
·46501661	'7'2'8'0'0'0'0'0 ↑	'76562829
·46457662	'7'2'9'0'0'0'0'0 ↑	'76662879
·46409100	'7'3'0'0'0'0'0'0 ↑	'76767463
·46362691	'7'3'1'0'0'0'0'0 ↑	'76867513
·46316328	'7'3'2'0'0'0'0'0 ↑	'76967563
·46270012	'7'3'3'0'0'0'0'0 ↑	'77067613
·46223742	'7'3'4'0'0'0'0'0 ↑	'77167663
·46177518	'7'3'5'0'0'0'0'0 ↑	'77267713
·46131340	'7'3'6'0'0'0'0'0 ↑	'77367763
·46085209	'7'3'7'0'0'0'0'0 ↑	'77467813
·46039124	'7'3'8'0'0'0'0'0 ↑	'77567863
·45993085	'7'3'9'0'0'0'0'0 ↑	'77667913
·45945009	'7'4'0'0'0'0'0'0 ↑	'77772497
·45899064	'7'4'1'0'0'0'0'0 ↑	'77872547
·45853165	'7'4'2'0'0'0'0'0 ↑	'77972597
·45807312	'7'4'3'0'0'0'0'0 ↑	'78072647
·45761505	'7'4'4'0'0'0'0'0 ↑	'78172697
·45715743	'7'4'5'0'0'0'0'0 ↑	'78272747
·45670028	'7'4'6'0'0'0'0'0 ↑	'78372797
·45624358	'7'4'7'0'0'0'0'0 ↑	'78472847
·45578733	'7'4'8'0'0'0'0'0 ↑	'78572897
·45533155	'7'4'9'0'0'0'0'0 ↑	'78672947
·45485559	'7'5'0'0'0'0'0'0 ↑	'78777531

2 = 69314718   10 = 230258509
4 = 138629436   100 = 460517018
8 = 207944154   1000 = 690775527

N. Nos.	D. Nos.	D. Logs.
·45440073	'7'5'1'0'0'0'0'0 ↑	'78877581
·45394633	'7'5'2'0'0'0'0'0 ↑	'78977631
·45349238	'7'5'3'0'0'0'0'0 ↑	'79077681
·45303889	'7'5'4'0'0'0'0'0 ↑	'79177731
·45258585	'7'5'5'0'0'0'0'0 ↑	'79277781
·45213326	'7'5'6'0'0'0'0'0 ↑	'79377831
·45168114	'7'5'7'0'0'0'0'0 ↑	'79477881
·45122946	'7'5'8'0'0'0'0'0 ↑	'79577931
·45077823	'7'5'9'0'0'0'0'0 ↑	'79677981
·45030703	'7'6'0'0'0'0'0'0 ↑	'79782565
·44985672	'7'6'1'0'0'0'0'0 ↑	'79882615
·44940686	'7'6'2'0'0'0'0'0 ↑	'79982665
·44895745	'7'6'3'0'0'0'0'0 ↑	'80082715
·44850849	'7'6'4'0'0'0'0'0 ↑	'80182765
·44806000	'7'6'5'0'0'0'0'0 ↑	'80282815
·44761192	'7'6'6'0'0'0'0'0 ↑	'80382865
·44716431	'7'6'7'0'0'0'0'0 ↑	'80482915
·44671715	'7'6'8'0'0'0'0'0 ↑	'80582965
·44627043	'7'6'9'0'0'0'0'0 ↑	'80683015
·44580396	'7'7'0'0'0'0'0'0 ↑	'80787599
·44535816	'7'7'1'0'0'0'0'0 ↑	'80887649
·44491280	'7'7'2'0'0'0'0'0 ↑	'80987699
·44446789	'7'7'3'0'0'0'0'0 ↑	'81087749
·44402342	'7'7'4'0'0'0'0'0 ↑	'81187799
·44357940	'7'7'5'0'0'0'0'0 ↑	'81287849
·44313582	'7'7'6'0'0'0'0'0 ↑	'81387899
·44269268	'7'7'7'0'0'0'0'0 ↑	'81487949
·44224999	'7'7'8'0'0'0'0'0 ↑	'81587999
·44180774	'7'7'9'0'0'0'0'0 ↑	'81688049
·44134592	'7'8'0'0'0'0'0'0 ↑	'81792633
·44090457	'7'8'1'0'0'0'0'0 ↑	'81892683
·44046367	'7'8'2'0'0'0'0'0 ↑	'81992733
·44002321	'7'8'3'0'0'0'0'0 ↑	'82092783
·43958319	'7'8'4'0'0'0'0'0 ↑	'82192833
·43914361	'7'8'5'0'0'0'0'0 ↑	'82292883
·43870447	'7'8'6'0'0'0'0'0 ↑	'82392933
·43826577	'7'8'7'0'0'0'0'0 ↑	'82492983
·43782749	'7'8'8'0'0'0'0'0 ↑	'82593033
·43738967	'7'8'9'0'0'0'0'0 ↑	'82693083
·43693246	'7'9'0'0'0'0'0'0 ↑	'82797667
·43649553	'7'9'1'0'0'0'0'0 ↑	'82897717
·43605903	'7'9'2'0'0'0'0'0 ↑	'82997767
·43562297	'7'9'3'0'0'0'0'0 ↑	'83097817
·43518735	'7'9'4'0'0'0'0'0 ↑	'83197867
·43475216	'7'9'5'0'0'0'0'0 ↑	'83297917
·43431741	'7'9'6'0'0'0'0'0 ↑	'83397967
·43388309	'7'9'7'0'0'0'0'0 ↑	'83497017
·43344921	'7'9'8'0'0'0'0'0 ↑	'83598067
·43301576	'7'9'9'0'0'0'0'0 ↑	'83698117
·43046721	'8'0'0'0'0'0'0'0 ↑	'84288413

2 = 69314718   10 = 230258509
4 = 138629436   100 = 460517018
8 = 207944154   1000 = 690775527

N. Nos.	D. Nos.	D. Logs.
·43004573	'8·0·1·0·0·0·0·0 ↑	'84388463
·42961569	'8·0·2·0·0·0·0·0 ↑	'84488513
·42918607	'8·0·3·0·0·0·0·0 ↑	'84588563
·42875689	'8·0·4·0·0·0·0·0 ↑	'84688613
·42832813	'8·0·5·0·0·0·0·0 ↑	'84788663
·42789980	'8·0·6·0·0·0·0·0 ↑	'84888713
·42747190	'8·0·7·0·0·0·0·0 ↑	'84988763
·42704443	'8·0·8·0·0·0·0·0 ↑	'85088813
·42661739	'8·0·9·0·0·0·0·0 ↑	'85188863
·42616254	'8·1·0·0·0·0·0·0 ↑	'85293447
·42573638	'8·1·1·0·0·0·0·0 ↑	'85393497
·42531064	'8·1·2·0·0·0·0·0 ↑	'85493547
·42488533	'8·1·3·0·0·0·0·0 ↑	'85593597
·42446044	'8·1·4·0·0·0·0·0 ↑	'85693647
·42403598	'8·1·5·0·0·0·0·0 ↑	'85793697
·42361194	'8·1·6·0·0·0·0·0 ↑	'85893747
·42318833	'8·1·7·0·0·0·0·0 ↑	'85993797
·42276514	'8·1·8·0·0·0·0·0 ↑	'86093847
·42234237	'8·1·9·0·0·0·0·0 ↑	'86193897
·42190091	'8·2·0·0·0·0·0·0 ↑	'86294881
·42147901	'8·2·1·0·0·0·0·0 ↑	'86394931
·42105753	'8·2·2·0·0·0·0·0 ↑	'86494981
·42063647	'8·2·3·0·0·0·0·0 ↑	'86595031
·42021583	'8·2·4·0·0·0·0·0 ↑	'86695081
·41979562	'8·2·5·0·0·0·0·0 ↑	'86795131
·41937583	'8·2·6·0·0·0·0·0 ↑	'86895181
·41895646	'8·2·7·0·0·0·0·0 ↑	'86995231
·41853750	'8·2·8·0·0·0·0·0 ↑	'87095281
·41811896	'8·2·9·0·0·0·0·0 ↑	'87195331
·41768190	'8·3·0·0·0·0·0·0 ↑	'87303515
·41726422	'8·3·1·0·0·0·0·0 ↑	'87403565
·41684696	'8·3·2·0·0·0·0·0 ↑	'87503615
·41643011	'8·3·3·0·0·0·0·0 ↑	'87603665
·41601368	'8·3·4·0·0·0·0·0 ↑	'87703715
·41559767	'8·3·5·0·0·0·0·0 ↑	'87803765
·41518207	'8·3·6·0·0·0·0·0 ↑	'87903815
·41476689	'8·3·7·0·0·0·0·0 ↑	'88003865
·41435212	'8·3·8·0·0·0·0·0 ↑	'88103915
·41393780	'8·3·9·0·0·0·0·0 ↑	'88203965
·41350508	'8·4·0·0·0·0·0·0 ↑	'88308549
·41309157	'8·4·1·0·0·0·0·0 ↑	'88408599
·41267848	'8·4·2·0·0·0·0·0 ↑	'88508649
·41226580	'8·4·3·0·0·0·0·0 ↑	'88608699
·41185353	'8·4·4·0·0·0·0·0 ↑	'88708749
·41144168	'8·4·5·0·0·0·0·0 ↑	'88808799
·41103024	'8·4·6·0·0·0·0·0 ↑	'88908849
·41061921	'8·4·7·0·0·0·0·0 ↑	'89008899
·41020859	'8·4·8·0·0·0·0·0 ↑	'89108949
·40979838	'8·4·9·0·0·0·0·0 ↑	'89208999
·40937003	'8·5·0·0·0·0·0·0 ↑	'89313583

2 = 69314718	10 = 230258509
4 = 138629436	100 = 460517018
8 = 207944154	1000 = 690775527

N. Nos.	D. Nos.	D. Logs.
·40896066	·8·5·1·0·0·0·0·0 ↑	·89413633
·40855170	·8·5·2·0·0·0·0·0 ↑	·89513683
·40814315	·8·5·3·0·0·0·0·0 ↑	·89613733
·40773501	·8·5·4·0·0·0·0·0 ↑	·89713783
·40732728	·8·5·5·0·0·0·0·0 ↑	·89813833
·40691995	·8·5·6·0·0·0·0·0 ↑	·89913883
·40651303	·8·5·7·0·0·0·0·0 ↑	·90013933
·40610652	·8·5·8·0·0·0·0·0 ↑	·90113983
·40570041	·8·5·9·0·0·0·0·0 ↑	·90214033
·40527633	·8·6·0·0·0·0·0·0 ↑	·90318617
·40487105	·8·6·1·0·0·0·0·0 ↑	·90418667
·40446618	·8·6·2·0·0·0·0·0 ↑	·90518717
·40406171	·8·6·3·0·0·0·0·0 ↑	·90618767
·40365765	·8·6·4·0·0·0·0·0 ↑	·90718817
·40325399	·8·6·5·0·0·0·0·0 ↑	·90818867
·40285074	·8·6·6·0·0·0·0·0 ↑	·90918917
·40244789	·8·6·7·0·0·0·0·0 ↑	·91018967
·40204544	·8·6·8·0·0·0·0·0 ↑	·91119017
·40164339	·8·6·9·0·0·0·0·0 ↑	·91219067
·40122357	·8·7·0·0·0·0·0·0 ↑	·91323651
·40082235	·8·7·1·0·0·0·0·0 ↑	·91423701
·40042153	·8·7·2·0·0·0·0·0 ↑	·91523751
·40002111	·8·7·3·0·0·0·0·0 ↑	·91623801
·39962109	·8·7·4·0·0·0·0·0 ↑	·91723851
·39922147	·8·7·5·0·0·0·0·0 ↑	·91823901
·39882225	·8·7·6·0·0·0·0·0 ↑	·91923951
·39842343	·8·7·7·0·0·0·0·0 ↑	·92024001
·39802501	·8·7·8·0·0·0·0·0 ↑	·92124051
·39762698	·8·7·9·0·0·0·0·0 ↑	·92224101
·39721133	·8·8·0·0·0·0·0·0 ↑	·92328685
·39681412	·8·8·1·0·0·0·0·0 ↑	·92428735
·39641731	·8·8·2·0·0·0·0·0 ↑	·92528785
·39602089	·8·8·3·0·0·0·0·0 ↑	·92628835
·39562487	·8·8·4·0·0·0·0·0 ↑	·92728885
·39522925	·8·8·5·0·0·0·0·0 ↑	·92828935
·39483402	·8·8·6·0·0·0·0·0 ↑	·92928985
·39483402	·8·8·7·0·0·0·0·0 ↑	·93029035
·39404475	·8·8·8·0·0·0·0·0 ↑	·93129085
·39365071	·8·8·9·0·0·0·0·0 ↑	·93229135
·39323922	·8·9·0·0·0·0·0·0 ↑	·93333719
·39284598	·8·9·1·0·0·0·0·0 ↑	·93433769
·39245313	·8·9·2·0·0·0·0·0 ↑	·93533819
·39206068	·8·9·3·0·0·0·0·0 ↑	·93633869
·39166862	·8·9·4·0·0·0·0·0 ↑	·93733919
·39127695	·8·9·5·0·0·0·0·0 ↑	·93833969
·39088567	·8·9·6·0·0·0·0·0 ↑	·93934019
·39049478	·8·9·7·0·0·0·0·0 ↑	·94034069
·39010430	·8·9·8·0·0·0·0·0 ↑	·94134119
·39971419	·8·9·9·0·0·0·0·0 ↑	·94234169
·38742049	·9·0·0·0·0·0·0·0·0 ↑	·94824464

2 = 69314718	10 = 230258509
4 = 138629436	100 = 460517018
8 = 207944154	1000 = 690775527

N. Nos.	D. Nos.	D. Logs.
·38703307	'9'0'1'0'0'0'0'0'0 ↑	'94924514
·38664604	'9'0'2'0'0'0'0'0'0 ↑	'95024564
·38625939	'9'0'3'0'0'0'0'0'0 ↑	'95124614
·38587313	'9'0'4'0'0'0'0'0'0 ↑	'95224664
·38548726	'9'0'5'0'0'0'0'0'0 ↑	'95324714
·38510177	'9'0'6'0'0'0'0'0'0 ↑	'95424764
·38471667	'9'0'7'0'0'0'0'0'0 ↑	'95524814
·38433195	'9'0'8'0'0'0'0'0'0 ↑	'95624864
·38394762	'9'0'9'0'0'0'0'0'0 ↑	'95724914
·38354629	'9'1'0'0'0'0'0'0'0 ↑	'95829498
·38316274	'9'1'1'0'0'0'0'0'0 ↑	'95929548
·38277959	'9'1'2'0'0'0'0'0'0 ↑	'96029598
·38239680	'9'1'3'0'0'0'0'0'0 ↑	'96129648
·38201440	'9'1'4'0'0'0'0'0'0 ↑	'96229698
·38163239	'9'1'5'0'0'0'0'0'0 ↑	'96329748
·38124075	'9'1'6'0'0'0'0'0'0 ↑	'96429798
·38086950	'9'1'7'0'0'0'0'0'0 ↑	'96529848
·38048863	'9'1'8'0'0'0'0'0'0 ↑	'96629898
·38010814	'9'1'9'0'0'0'0'0'0 ↑	'96729948
·37971083	'9'2'0'0'0'0'0'0'0 ↑	'96834533
·37933112	'9'2'1'0'0'0'0'0'0 ↑	'96934583
·37895179	'9'2'2'0'0'0'0'0'0 ↑	'97034633
·37857284	'9'2'3'0'0'0'0'0'0 ↑	'97134683
·37819427	'9'2'4'0'0'0'0'0'0 ↑	'97234733
·37781608	'9'2'5'0'0'0'0'0'0 ↑	'97334783
·37743824	'9'2'6'0'0'0'0'0'0 ↑	'97434833
·37706080	'9'2'7'0'0'0'0'0'0 ↑	'97534883
·37668374	'9'2'8'0'0'0'0'0'0 ↑	'97634933
·37630706	'9'2'9'0'0'0'0'0'0 ↑	'97734983
·37591372	'9'3'0'0'0'0'0'0'0 ↑	'97839567
·37553781	'9'3'1'0'0'0'0'0'0 ↑	'97939617
·37516227	'9'3'2'0'0'0'0'0'0 ↑	'98039667
·37478711	'9'3'3'0'0'0'0'0'0 ↑	'98139717
·37441232	'9'3'4'0'0'0'0'0'0 ↑	'98239767
·37403790	'9'3'5'0'0'0'0'0'0 ↑	'98339817
·37366386	'9'3'6'0'0'0'0'0'0 ↑	'98439867
·37329020	'9'3'7'0'0'0'0'0'0 ↑	'98539917
·37291691	'9'3'8'0'0'0'0'0'0 ↑	'98639967
·37254399	'9'3'9'0'0'0'0'0'0 ↑	'98740017
·37215458	'9'4'0'0'0'0'0'0'0 ↑	'98844601
·37178243	'9'4'1'0'0'0'0'0'0 ↑	'98944651
·37141065	'9'4'2'0'0'0'0'0'0 ↑	'99044701
·37103924	'9'4'3'0'0'0'0'0'0 ↑	'99144751
·37066820	'9'4'4'0'0'0'0'0'0 ↑	'99244801
·37029753	'9'4'5'0'0'0'0'0'0 ↑	'99344851
·36992723	'9'4'6'0'0'0'0'0'0 ↑	'99444901
·36955730	'9'4'7'0'0'0'0'0'0 ↑	'99544951
·36918774	'9'4'8'0'0'0'0'0'0 ↑	'99645001
·36881855	'9'4'9'0'0'0'0'0'0 ↑	'99745051
·36843303	'9'5'0'0'0'0'0'0'0 ↑	'99849635

2 = 69314718	10 = 230258509
4 = 138629436	100 = 460517018
8 = 207944154	1000 = 690775527

N. Nos.	D. Nos.	D. Logs.
·36806460	'9'5'1'0'0'0'0'0 ↑	'99949685
·36769654	'9'5'2'0'0'0'0'0 ↑	'100049735
·36732884	'9'5'3'0'0'0'0'0 ↑	'100149785
·36696151	'9'5'4'0'0'0'0'0 ↑	'100249835
·36659455	'9'5'5'0'0'0'0'0 ↑	'100349885
·36622796	'9'5'6'0'0'0'0'0 ↑	'100449935
·36586173	'9'5'7'0'0'0'0'0 ↑	'100549985
·36549587	'9'5'8'0'0'0'0'0 ↑	'100650035
·36513037	'9'5'9'0'0'0'0'0 ↑	'100750085
·36474870	'9'6'0'0'0'0'0'0 ↑	'100854669
·36438395	'9'6'1'0'0'0'0'0 ↑	'100954719
·36401957	'9'6'2'0'0'0'0'0 ↑	'101054769
·36365555	'9'6'3'0'0'0'0'0 ↑	'101154819
·36329189	'9'6'4'0'0'0'0'0 ↑	'101254869
·36292860	'9'6'5'0'0'0'0'0 ↑	'101354919
·36256567	'9'6'6'0'0'0'0'0 ↑	'101454969
·36220310	'9'6'7'0'0'0'0'0 ↑	'101555019
·36184090	'9'6'8'0'0'0'0'0 ↑	'101655069
·36147906	'9'6'9'0'0'0'0'0 ↑	'101755119
·36110121	'9'7'0'0'0'0'0'0 ↑	'101859703
·36074011	'9'7'1'0'0'0'0'0 ↑	'101959753
·36037937	'9'7'2'0'0'0'0'0 ↑	'102059803
·36001899	'9'7'3'0'0'0'0'0 ↑	'102159853
·35965897	'9'7'4'0'0'0'0'0 ↑	'102259903
·35929931	'9'7'5'0'0'0'0'0 ↑	'102359953
·35894001	'9'7'6'0'0'0'0'0 ↑	'102460003
·35858107	'9'7'7'0'0'0'0'0 ↑	'102560053
·35822249	'9'7'8'0'0'0'0'0 ↑	'102660103
·35786427	'9'7'9'0'0'0'0'0 ↑	'102760153
·35749020	'9'8'0'0'0'0'0'0 ↑	'102864737
·35713271	'9'8'1'0'0'0'0'0 ↑	'102964787
·35677558	'9'8'2'0'0'0'0'0 ↑	'103064837
·35641880	'9'8'3'0'0'0'0'0 ↑	'103164887
·35606238	'9'8'4'0'0'0'0'0 ↑	'103264937
·35570632	'9'8'5'0'0'0'0'0 ↑	'103364987
·35535061	'9'8'6'0'0'0'0'0 ↑	'103465037
·35499526	'9'8'7'0'0'0'0'0 ↑	'103565087
·35464026	'9'8'8'0'0'0'0'0 ↑	'103665137
·35428563	'9'8'9'0'0'0'0'0 ↑	'103765187
·35391530	'9'9'0'0'0'0'0'0 ↑	'103869771
·35356138	'9'9'1'0'0'0'0'0 ↑	'103969821
·35320782	'9'9'2'0'0'0'0'0 ↑	'104069871
·35285461	'9'9'3'0'0'0'0'0 ↑	'104169921
·35250176	'9'9'4'0'0'0'0'0 ↑	'104269971
·35214926	'9'9'5'0'0'0'0'0 ↑	'104370021
·35179711	'9'9'6'0'0'0'0'0 ↑	'104470071
·35144531	'9'9'7'0'0'0'0'0 ↑	'104570121
·35109387	'9'9'8'0'0'0'0'0 ↑	'104670171
·35074277	'9'9'9'0'0'0'0'0 ↑	'104770221
·34867844	'10'0'0'0'0'0'0'0'0 ↑	'105360516

2= 69314718	10=230258509
4=138629436	100=460517018
8=207944154	1000=690775527

N. Nos.	D. Nos.	D. Logs.
·34832976	'10'0'1'0'0'0'0'0 ↑	'105460566
·34798143	'10'0'2'0'0'0'0'0 ↑	'105560616
·34763345	'10'0'3'0'0'0'0'0 ↑	'105660666
·34728582	'10'0'4'0'0'0'0'0 ↑	'105760716
·34693853	'10'0'5'0'0'0'0'0 ↑	'105860766
·34659159	'10'0'6'0'0'0'0'0 ↑	'105960816
·34624500	'10'0'7'0'0'0'0'0 ↑	'106060866
·34589875	'10'0'8'0'0'0'0'0 ↑	'106160916
·34555285	'10'0'9'0'0'0'0'0 ↑	'106260966
·34519166	'10'1'0'0'0'0'0'0 ↑	'106365550
·34484647	'10'1'1'0'0'0'0'0 ↑	'106465600
·34450162	'10'1'2'0'0'0'0'0 ↑	'106565650
·34415712	'10'1'3'0'0'0'0'0 ↑	'106665700
·34381296	'10'1'4'0'0'0'0'0 ↑	'106765750
·34346915	'10'1'5'0'0'0'0'0 ↑	'106865800
·34312568	'10'1'6'0'0'0'0'0 ↑	'106965850
·34278255	'10'1'7'0'0'0'0'0 ↑	'107065900
·34243977	'10'1'8'0'0'0'0'0 ↑	'107165950
·34209734	'10'1'9'0'0'0'0'0 ↑	'107266000
·34173974	'10'2'0'0'0'0'0'0 ↑	'107370584
·34139800	'10'2'1'0'0'0'0'0 ↑	'107470634
·34105660	'10'2'2'0'0'0'0'0 ↑	'107570684
·34071554	'10'2'3'0'0'0'0'0 ↑	'107670734
·34037482	'10'2'4'0'0'0'0'0 ↑	'107770784
·34003445	'10'2'5'0'0'0'0'0 ↑	'107870834
·33969442	'10'2'6'0'0'0'0'0 ↑	'107970884
·33935473	'10'2'7'0'0'0'0'0 ↑	'108070934
·33901538	'10'2'8'0'0'0'0'0 ↑	'108170984
·33867636	'10'2'9'0'0'0'0'0 ↑	'108271034
·33832234	'10'3'0'0'0'0'0'0 ↑	'108375618
·33798402	'10'3'1'0'0'0'0'0 ↑	'108475668
·33764604	'10'3'2'0'0'0'0'0 ↑	'108575718
·33730839	10'3'3'0'0'0'0'0 ↑	'108675768
·33697108	'10'3'4'0'0'0'0'0 ↑	'108775818
·33663411	'10'3'5'0'0'0'0'0 ↑	'108875868
·33629748	'10'3'6'0'0'0'0'0 ↑	'108975918
·33596118	'10'3'7'0'0'0'0'0 ↑	'109075968
·33562522	'10'3'8'0'0'0'0'0 ↑	'109176018
·33528959	'10'3'9'0'0'0'0'0 ↑	'109276068
·33493912	'10'4'0'0'0'0'0'0 ↑	'109380652
·33460418	'10'4'1'0'0'0'0'0 ↑	'109480702
·33426958	'10'4'2'0'0'0'0'0 ↑	'109580752
·33393531	'10'4'3'0'0'0'0'0 ↑	'109680802
·33360137	'10'4'4'0'0'0'0'0 ↑	'109780852
·33326777	'10'4'5'0'0'0'0'0 ↑	'109880902
·33293450	'10'4'6'0'0'0'0'0 ↑	'109980952
·33260157	'10'4'7'0'0'0'0'0 ↑	'110081002
·33226897	'10'4'8'0'0'0'0'0 ↑	'110181052
·33193670	'10'4'9'0'0'0'0'0 ↑	'110281102
·33158973	'10'5'0'0'0'0'0'0 ↑	'110385686

2 = 69314718 10 = 230258509
4 = 138629436 100 = 460517018
8 = 207944154 1000 = 690775527

N. Nos.	D. Nos.	D. Logs.
·33125814	'10'5'1'0'0'0'0'0 ↑	'110485736
·33192688	'10'5'2'0'0'0'0'0 ↑	'110585786
·33059595	'10'5'3'0'0'0'0'0 ↑	'110685836
·33026535	'10'5'4'0'0'0'0'0 ↑	'110785886
·32993508	'10'5'5'0'0'0'0'0 ↑	'110885936
·32960514	'10'5'6'0'0'0'0'0 ↑	'110985986
·32927553	'10'5'7'0'0'0'0'0 ↑	'111086036
·32894625	'10'5'8'0'0'0'0'0 ↑	'111186086
·32861730	'10'5'9'0'0'0'0'0 ↑	'111286136
·32827383	'10'6'0'0'0'0'0'0 ↑	'111390720
·32794556	'10'6'1'0'0'0'0'0 ↑	'111490770
·32761761	'10'6'2'0'0'0'0'0 ↑	'111590820
·32728999	'10'6'3'0'0'0'0'0 ↑	'111690870
·32696270	'10'6'4'0'0'0'0'0 ↑	'111790920
·32663574	'10'6'5'0'0'0'0'0 ↑	'111890970
·32630910	'10'6'6'0'0'0'0'0 ↑	'111991020
·32598280	'10'6'7'0'0'0'0'0 ↑	'112091070
·32565682	10'6'8'0'0'0'0'0 ↑	'112191120
·32533116	'10'6'9'0'0'0'0'0 ↑	'112291170
·32499109	0'7'0'0'0'0'0'0 ↑	'112395754
·32466610	10'7'1'0'0'0'0'0 ↑	'112495804
·32434143	'10'7'2'0'0'0'0'0 ↑	'112595854
·32401709	'10'7'3'0'0'0'0'0 ↑	'112695904
·32369307	'10'7'4'0'0'0'0'0 ↑	'112795954
·32336938	'10'7'5'0'0'0'0'0 ↑	'112896004
·32304601	'10'7'6'0'0'0'0'0 ↑	'112996054
·32272296	'10'7'7'0'0'0'0'0 ↑	'113096104
·32240024	'10'7'8'0'0'0'0'0 ↑	'113196154
·32207784	'10'7'9'0'0'0'0'0 ↑	'113296204
·32174118	'10'8'0'0'0'0'0'0 ↑	'113406788
·32141944	'10'8'1'0'0'0'0'0 ↑	'113500838
·32109802	'10'8'2'0'0'0'0'0 ↑	'113600888
·32077692	'10'8'3'0'0'0'0'0 ↑	'113700938
·32045614	'10'8'4'0'0'0'0'0 ↑	'113800988
·32013568	'10'8'5'0'0'0'0'0 ↑	'113901038
·31981554	'10'8'6'0'0'0'0'0 ↑	'114001088
·31949572	'10'8'7'0'0'0'0'0 ↑	'114101138
·31917622	'10'8'8'0'0'0'0'0 ↑	'114201188
·31885704	'10'8'9'0'0'0'0'0 ↑	'114301238
·31852377	'10'9'0'0'0'0'0'0 ↑	'114405822
·31820525	'10'9'1'0'0'0'0'0 ↑	'114505872
·31788704	'10'9'2'0'0'0'0'0 ↑	'114605922
·31756915	'10'9'3'0'0'0'0'0 ↑	'114705972
·31725158	'10'9'4'0'0'0'0'0 ↑	'114806022
·31693433	'10'9'5'0'0'0'0'0 ↑	'114906072
·31661740	'10'9'6'0'0'0'0'0 ↑	'115006122
·31630078	10'9'7'0'0'0'0'0 ↑	'115106172
·31598448	'10'9'8'0'0'0'0'0 ↑	'115206222
·31566850	'10'9'9'0'0'0'0'0 ↑	'115306272
·31381060	'11'0'0'0'0'0'0'0 ↑	'115896568

2 = 69314718 10 = 230258509
4 = 138629436 100 = 460517018
8 = 207944154 1000 = 690775527

N. Nos.	D. Nos.	D. Logs.
·31349679	'11·0·1·0·0·0·0·0 ↑	'115996618
·31318329	'11·0·2·0·0·0·0·0 ↑	'116096668
·31287011	'11·0·3·0·0·0·0·0 ↑	'116196718
·31255724	'11·0·4·0·0·0·0·0 ↑	'116296768
·31224468	'11·0·5·0·0·0·0·0 ↑	'116396818
·31193244	'11·0·6·0·0·0·0·0 ↑	'116496868
·31162051	'11·0·7·0·0·0·0·0 ↑	'116596918
·31130889	'11·0·8·0·0·0·0·0 ↑	'116696968
·31099758	'11·0·9·0·0·0·0·0 ↑	'116797018
·31067249	'11·1·0·0·0·0·0·0 ↑	'116901602
·31036182	'11·1·1·0·0·0·0·0 ↑	'117001652
·31005146	'11·1·2·0·0·0·0·0 ↑	'117101702
·30974141	'11·1·3·0·0·0·0·0 ↑	'117201752
·30943167	'11·1·4·0·0·0·0·0 ↑	'117301802
·30912224	'11·1·5·0·0·0·0·0 ↑	'117401852
·30881312	'11·1·6·0·0·0·0·0 ↑	'117501902
·30850431	'11·1·7·0·0·0·0·0 ↑	'117601952
·30819581	'11·1·8·0·0·0·0·0 ↑	'117702002
·30788761	'11·1·9·0·0·0·0·0 ↑	'117802052
·30756577	'11·2·0·0·0·0·0·0 ↑	'117906636
·30725820	'11·2·1·0·0·0·0·0 ↑	'118006686
·30695094	'11·2·2·0·0·0·0·0 ↑	'118106736
·30664399	'11·2·3·0·0·0·0·0 ↑	'118206786
·30633735	'11·2·4·0·0·0·0·0 ↑	'118306836
·30603101	'11·2·5·0·0·0·0·0 ↑	'118406886
·30572498	'11·2·6·0·0·0·0·0 ↑	'118506936
·30541926	'11·2·7·0·0·0·0·0 ↑	'118606986
·30511384	'11·2·8·0·0·0·0·0 ↑	'118707036
·30480873	'11·2·9·0·0·0·0·0 ↑	'118807086
·30449011	'11·3·0·0·0·0·0·0 ↑	'118911670
·30418562	'11·3·1·0·0·0·0·0 ↑	'119011720
·30388143	'11·3·2·0·0·0·0·0 ↑	'119111770
·30357755	'11·3·3·0·0·0·0·0 ↑	'119211820
·30327397	'11·3·4·0·0·0·0·0 ↑	'119311870
·30297072	'11·3·5·0·0·0·0·0 ↑	'119411920
·30266775	'11·3·6·0·0·0·0·0 ↑	'119511970
·30236508	'11·3·7·0·0·0·0·0 ↑	'119612020
·30206271	'11·3·8·0·0·0·0·0 ↑	'119712070
·30176065	'11·3·9·0·0·0·0·0 ↑	'119812120
·30144521	'11·4·0·0·0·0·0·0 ↑	'119916704
·30114377	'11·4·1·0·0·0·0·0 ↑	'120016754
·30084263	'11·4·2·0·0·0·0·0 ↑	'120116804
·30054179	'11·4·3·0·0·0·0·0 ↑	'120216854
·30024125	'11·4·4·0·0·0·0·0 ↑	'120316904
·29994101	'11·4·5·0·0·0·0·0 ↑	'120416954
·29964107	'11·4·6·0·0·0·0·0 ↑	'120517004
·29934143	'11·4·7·0·0·0·0·0 ↑	'120617054
·29916114	'11·4·7·6·0·2·3·9 ↑	'120677293
·29886198	'11·4·8·0·0·0·0·0 ↑	'120717104

2 = 69314718	·10 = 230258509
4 = 138629436	100 = 460517018
8 = 207944154	1000 = 690775527

'1'1'4'7'6'0'2'3'9 $\uparrow$ 10·=2·9916114=
$\uparrow$ 11,4,7,6,0,2,3,9, ; hence the dual logarithm of any given number may be obtained from these tables, and the log. of 10,=230258509, ·

$\downarrow$, (2991·6114)='120677293+4 $\downarrow$, (10)
4 $\downarrow$, (10)= 921034036,
$\downarrow$, (·29916114)='120677293

800356753,=$\downarrow$, (2991·6114)

Although we employ multiples of $\downarrow$, (10) here and in other places, as well as in the introduction to these tables, in order to convert $\downarrow$, (·29916114) to $\downarrow$, (2991·6114) or to $\downarrow$, (·0029916114) &c. or to convert $\downarrow$, (1·378201) to $\downarrow$, (1378·201) ; $\downarrow$, (·01378201) ; &c., yet in practice such multiples are seldom required (see Chapters V. and VI.). For example, multiply ·00086194541 ; 1378·201 ; and 2991·6114 continually together and divide the product by the product of ·78539816 and 1865. *Ans.* 2·42621194.

A	B	C	Add
'4	$\downarrow$, (·86194541)=	'14856338	'14856338
3,	$\downarrow$, (1·378201) =	32077903,	$\bar{1}$67922097 ar. co.
3,	$\downarrow$, (·29916114)=	'120677293	'120677293
0	$\downarrow$, (·78539816)=	'24156447	$\bar{1}$75843553 ar. co.
3,	$\downarrow$, (1·865) =	62326108,	'62326108
'1			'141625389
			230258509,=$\downarrow$,(10)

$\downarrow$ (2·42621194)= 88633120,

In practice the numbers C, taken from the tables, do not require to be set down. Nor is it necessary to write down A, which is only to show how many places of figures the decimal point should be removed to the right or left in B to produce the given numbers to be operated upon.

The work may also stand thus

Add
$\bar{1}$85143662 ar. co.
32077903,
$\bar{1}$879322707 ar. co.
$\bar{1}$24156447,
$\bar{1}$37673892 ar. co.
230258509,

$\downarrow$ (2·42621194)=88633120, as above.

To find where the ascending and descending branches coincide, or where the dual numbers are composed of the same digits and the corresponding natural numbers expressed by the same figures, let x be the required power of both 1·1 and ·9; then

$$(1{\cdot}1)^x = 10({\cdot}9)^x$$

$$\therefore\ x \downarrow,(1{\cdot}1) = \downarrow,(10) + x \downarrow,({\cdot}9)$$

$$\therefore\ x = \frac{\downarrow,(10)}{\downarrow,(1{\cdot}1) - \downarrow,({\cdot}9)} = \frac{230258509}{20067070,} = 11{\cdot}4 \text{ \&c.}$$

Again, let y be the required power of 1·01 and ·99 the next bases in succession; then

$$(1{\cdot}1)^{11}(1{\cdot}01)^y = 10({\cdot}9)^{11}({\cdot}99)^y$$

$$\therefore\ 11 \downarrow,(1{\cdot}1) + y \downarrow,(1{\cdot}01) = \downarrow,(10) + 11 \downarrow,({\cdot}9) + y \downarrow,({\cdot}99)$$

$$\therefore\ y = \frac{\downarrow,(10) - 11 \downarrow,(1{\cdot}1) + 11 \downarrow,({\cdot}9)}{\downarrow,(1{\cdot}01) - \downarrow,({\cdot}99)} = \frac{9520743}{2000067} = 4,7,6,0,$$

$$\text{Again } \frac{\downarrow,(10) - \downarrow,11,4,7,6,0, + '11'4'7'6'0'\uparrow}{\downarrow,{}^{6}1, - '1_6'\uparrow} = \frac{475,}{200} = 2,3,8.$$

$$\therefore\ \downarrow 11,4,7,6,0,2,3,8,5, \text{ and } '11'4'7'6\,0'2'3'8'5 \uparrow$$

are the required dual numbers, and 2·99161136 and ·299161136 the corresponding natural numbers.

↓ THE END. ↑

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